

Answer the following questions :

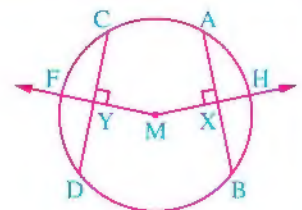
1 Choose the correct answer from those given :

- 1 The slope of the straight line $3x + 2y = 1$ is
 (a) $\frac{2}{3}$ (b) $-\frac{3}{2}$ (c) $-\frac{2}{3}$ (d) $\frac{3}{2}$
- 2 M and N are two intersecting circles , their radii lengths are 3 cm. and 5 cm. , then $MN \in$
 (a) $]8, \infty[$ (b) $]3, 5[$ (c) $]0, 2[$ (d) $]2, 8[$
- 3 The measurement of any angle of the regular hexagon is
 (a) 90° (b) 108° (c) 120° (d) 135°
- 4 ABCD is a cyclic quadrilateral , $m(\angle A) = 70^\circ$, then $m(\angle C)$ equals
 (a) 25° (b) 20° (c) 110° (d) 100°
- 5 In ΔABC , if $(AB)^2 = (AC)^2 + (BC)^2$, then $\angle B$ is
 (a) acute. (b) obtuse. (c) right. (d) reflex.
- 6 The measure of the inscribed angle drawn in a semicircle equals
 (a) 130° (b) 90° (c) 50° (d) 180°

2 [a] In the opposite figure :

\overline{AB} and \overline{CD} are two chords equal in length in the circle M
 $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$

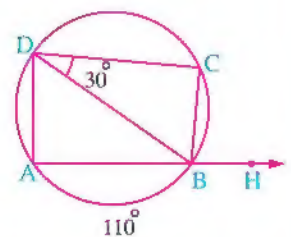
Prove that : $HX = FY$



[b] In the opposite figure :

$H \in \overline{AB}$, $m(\widehat{AB}) = 110^\circ$
 $m(\angle CDB) = 30^\circ$

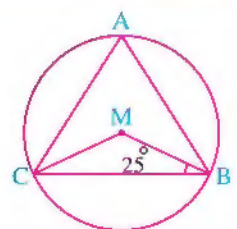
Find : $m(\angle HBC)$



3 [a] In the opposite figure :

ABC is a triangle drawn in the circle M
 $m(\angle MBC) = 25^\circ$

Find : $m(\angle BAC)$

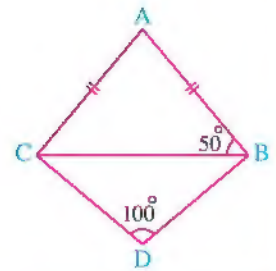


[b] In the opposite figure :

$$AB = AC, m(\angle D) = 100^\circ$$

$$, m(\angle ABC) = 50^\circ$$

Prove that : ABDC is a cyclic quadrilateral.



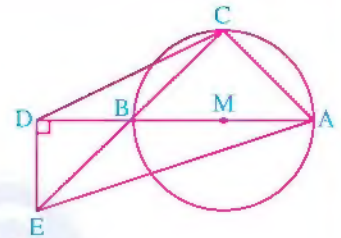
4 [a] In the opposite figure :

\overline{AB} is a diameter in the circle M

, $D \in \overline{AB}$, $D \notin \overline{AB}$, $\overline{DE} \perp \overline{AB}$

, $C \in \widehat{AB}$, $\overline{CB} \cap \overline{DE} = \{E\}$

Prove that : ACDE is a cyclic quadrilateral



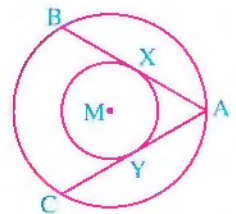
[b] In the opposite figure :

Two concentric circles of centre M

, \overline{AB} and \overline{AC} are two chords in the greater circle

and tangents to the smaller circle at X and Y respectively.

Prove that : $AB = AC$



5 [a] In the opposite figure :

M and N are two intersecting circles at A and B

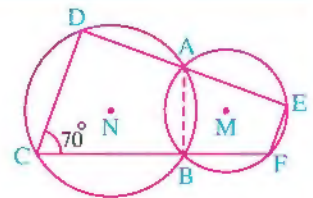
, \overline{AD} is drawn to intersect the circle M at E and

the circle N at D , \overline{AB} is drawn to intersect the circle M at

F and the circle N at C , $m(\angle BCD) = 70^\circ$

1 Find : $m(\angle EFB)$

2 Prove that : $\overline{CD} \parallel \overline{EF}$



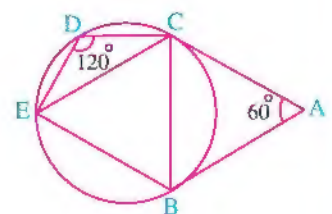
[b] In the opposite figure :

\overline{AB} and \overline{AC} are tangent-segments to the circle at B and C

, $m(\angle BAC) = 60^\circ$, $m(\angle CDE) = 120^\circ$

Prove that : **1** $\triangle BCE$ is an equilateral triangle.

2 $\overline{AC} \parallel \overline{BE}$



Answer the following questions :

1 Choose the correct answer from those given :

- 1 $\angle A$ and $\angle B$ are two complementary angles , $\angle B$ and $\angle C$ are two supplementary angles , $m(\angle A) = 30^\circ$, then $m(\angle C) = \dots\dots\dots^\circ$

(a) 30 (b) 60 (c) 90 (d) 120

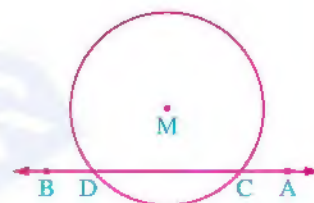
- 2 If the surface of the circle $M \cap$ the surface of the circle $N = \{A\}$ and the radius length of one of them equals 3 cm. and $MN = 8$ cm. , then the radius length of the other circle equals $\dots\dots\dots$ cm.

(a) 5 (b) 6 (c) 11 (d) 16

- 3 In the opposite figure :

$\overline{AB} \cap$ the surface of the circle $M = \dots\dots\dots$

(a) $\{C, D\}$ (b) \overline{CD}
(c) \overleftrightarrow{CD} (d) \emptyset



- 4 A circle can be drawn passing through the vertices of a $\dots\dots\dots$

(a) rhombus. (b) parallelogram. (c) trapezium. (d) rectangle.

- 5 The rhombus whose two diagonal lengths are 12 cm. and 16 cm. , then its side length equals $\dots\dots\dots$ cm.

(a) 6 (b) 8 (c) 10 (d) 20

- 6 In the opposite figure :

If the side length of the square = 10 cm.

, then the surface area of the circle = $\dots\dots\dots \text{cm}^2$

(a) 100π (b) 25π
(c) 50π (d) 40π

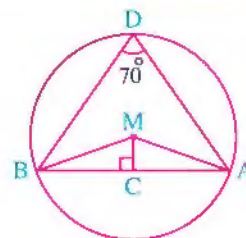


2 [a] In the opposite figure :

\overline{AB} is a chord in the circle M

, $\overline{MC} \perp \overline{AB}$, $m(\angle ADB) = 70^\circ$

Find : $m(\angle AMC)$

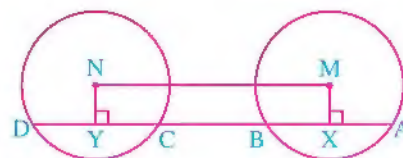


- [b] In the opposite figure :

M and N are two congruent circles

, $AB = CD$, $\overline{MX} \perp \overline{AB}$ and $\overline{NY} \perp \overline{CD}$

Prove that : The figure $MXYN$ is a rectangle.



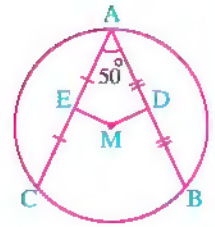
3 [a] In the opposite figure :

\overline{AB} and \overline{AC} are two chords

in the circle M, D is the midpoint of \overline{AB}

, E is the midpoint of \overline{AC} and $m(\angle BAC) = 50^\circ$

Find : $m(\angle DME)$



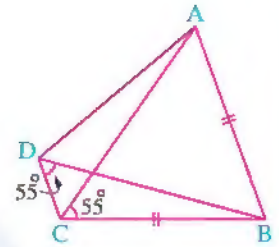
[b] In the opposite figure :

$AB = BC$

, $m(\angle ACB) = 55^\circ$

and $m(\angle BDC) = 55^\circ$

Prove that : The figure ABCD is a cyclic quadrilateral.



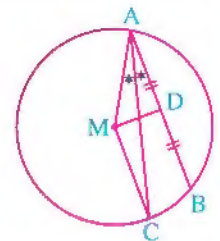
4 [a] In the opposite figure :

\overline{AB} is a chord in the circle M

, \overline{AC} bisects $\angle BAM$ and intersects the circle M at C

If D is the midpoint of \overline{AB}

, **prove that :** $\overline{DM} \perp \overline{CM}$



[b] \overline{AB} is a diameter in the circle M, \overline{AC} and \overline{BD} are two tangents to the circle M, \overline{CM} intersects the circle M at X and Y respectively and intersects \overline{BD} at E **Prove that :** $CX = YE$

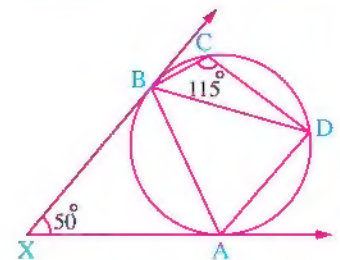
5 [a] In the opposite figure :

\overline{XA} and \overline{XB} are two tangents to the circle at A and B

, $m(\angle AXB) = 50^\circ$, $m(\angle DCB) = 115^\circ$

Prove that : ① \overline{AB} bisects $\angle DAX$

② $BD = BA$

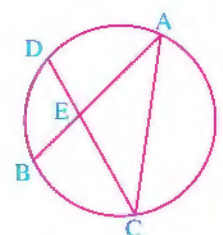


[b] In the opposite figure :

\overline{AB} and \overline{CD} are two equal chords in length in the circle

, $\overline{AB} \cap \overline{CD} = \{E\}$

Prove that : The triangle ACE is an isosceles triangle.



Answer the following questions :

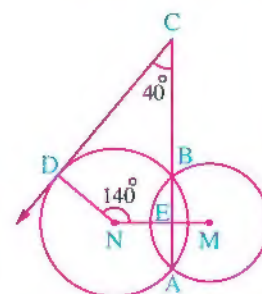
1 Choose the correct answer from those given :

- 1 The measure of the inscribed angle is the measure of the central angle subtended by the same arc.
(a) half (b) twice (c) quarter (d) third
- 2 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals the length of the hypotenuse.
(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{2}$ (d) 2
- 3 Two distant circles M and N with radii lengths 6 cm. and 8 cm. respectively , then MN 14 cm.
(a) < (b) > (c) = (d) \leq
- 4 The angle of measure 40° is the complemented angle of the angle of measure
(a) 320 (b) 140 (c) 60 (d) 50
- 5 The area of the rhombus with diagonal lengths 6 cm. , 8 cm. is cm^2
(a) 2 (b) 14 (c) 24 (d) 48
- 6 In the cyclic quadrilateral ABCD , if $m(\angle A) = \frac{1}{2} m(\angle C)$, then $m(\angle A) = \dots\dots\dots^\circ$
(a) 20 (b) 30 (c) 60 (d) 120

2 [a] In the opposite figure :

M and N are two intersecting circles at A and B
 $C \in \overrightarrow{AB}$, $\overrightarrow{AC} \cap \overrightarrow{MN} = \{E\}$
 $D \in \text{the circle N}$, $m(\angle DNM) = 140^\circ$
 and $m(\angle C) = 40^\circ$

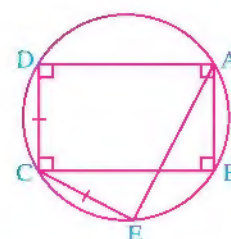
Prove that : \overrightarrow{CD} is a tangent to the circle N at D



[b] In the opposite figure :

ABCD is a rectangle inscribed in a circle
 , the chord \overline{CE} is drawn
 where $CE = CD$

Prove that : $AE = BC$



3 [a] State two cases of the cyclic quadrilateral.

[b] In the opposite figure :

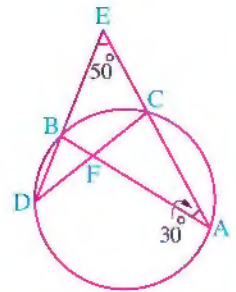
$$\overline{AB} \cap \overline{CD} = \{F\}, \overline{AC} \cap \overline{DB} = \{E\}$$

$$, m(\angle A) = 30^\circ$$

$$, m(\angle E) = 50^\circ$$

Find : **1** $m(\widehat{AD})$

2 $m(\angle AFD)$

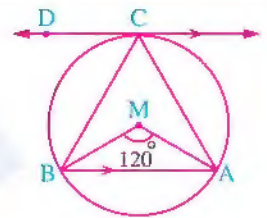


4 [a] In the opposite figure :

\overline{CD} is a tangent to the circle at C

$$, \overline{CD} \parallel \overline{AB}, m(\angle AMB) = 120^\circ$$

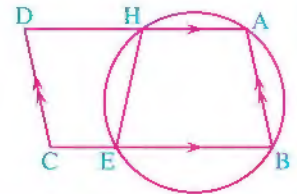
Prove that : The triangle CAB is an equilateral triangle.



[b] In the opposite figure :

ABCD is a parallelogram.

Prove that : HDCE is a cyclic quadrilateral.

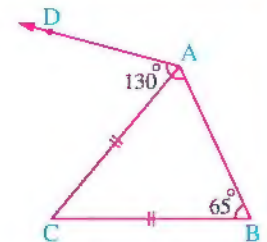


5 [a] In the opposite figure :

$$AC = BC, m(\angle ABC) = 65^\circ$$

$$, m(\angle DAB) = 130^\circ$$

Prove that : \overline{AD} is a tangent to the circle passing through the vertices of the triangle ABC



[b] In the opposite figure :

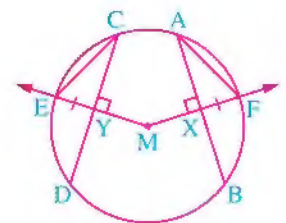
\overline{AB} and \overline{CD} are two chords in the circle M

$, \overline{MX} \perp \overline{AB}$ and intersects the circle at F

$, \overline{MY} \perp \overline{CD}$ and intersects the circle at E, $FX = EY$

Prove that : **1** $AB = CD$

2 $AF = CE$



Answers of model 1

1

1

1 b

2 d

3 c

4 c

5 a

6 b

2

[a] $\because AB = CD, \overline{MX} \perp \overline{AB}$
 $\overline{MY} \perp \overline{CD}$

$\therefore MX = MY$

$\therefore MH = MF = r$

$\therefore HX = FY$

(Q.E.D.)

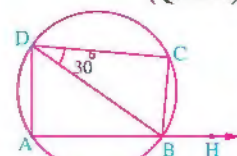


[b] $\because m(\angle ADB) = \frac{1}{2} m(\widehat{AB})$
 $= \frac{1}{2} \times 110^\circ$
 $= 55^\circ$

$\therefore ABCD$ is a cyclic quadrilateral. 110°

$\therefore m(\angle HBC) = m(\angle CDB) + m(\angle ADB)$

$= 30^\circ + 55^\circ = 85^\circ$ (The req.)



3

[a] In $\triangle BMC$:

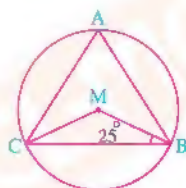
$\because MB = MC = r$

$\therefore m(\angle MCB)$
 $= m(\angle MBC) = 25^\circ$

$\therefore m(\angle BMC) = 180^\circ - (25^\circ + 25^\circ) = 130^\circ$

$\therefore m(\angle BAC) = \frac{1}{2} m(\angle BMC)$
 (inscribed and central angles subtended by \widehat{BC})

$\therefore m(\angle BAC) = \frac{1}{2} \times 130^\circ = 65^\circ$ (The req.)



[b] In $\triangle ABC$:

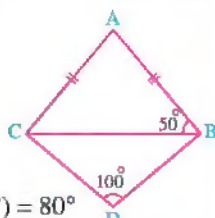
$\because AB = AC$

$\therefore m(\angle ACB) = m(\angle ABC)$
 $= 50^\circ$

$\therefore m(\angle A) = 180^\circ - (50^\circ + 50^\circ) = 80^\circ$

$\therefore m(\angle A) + m(\angle D) = 80^\circ + 100^\circ = 180^\circ$

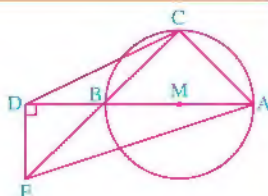
$\therefore ABDC$ is a cyclic quadrilateral. (Q.E.D.)



4

[a] $\because \overline{AB}$ is a diameter of the circle.

$\therefore m(\angle ACB) = 90^\circ$



$\therefore m(\angle ACE) = m(\angle ADE)$

and they are drawn on \overline{AE} and on one side of it

$\therefore ACDE$ is a cyclic quadrilateral. (Q.E.D.)

[b] Construction:

Draw $\overline{MX}, \overline{MY}$

Proof:

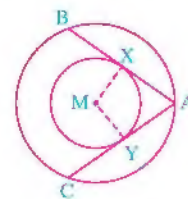
$\because \overline{AB}, \overline{AC}$ are two tangents to the smaller circle at X, Y respectively

$\therefore \overline{MX} \perp \overline{AB}, \overline{MY} \perp \overline{AC}$

$\therefore MX = MY = r$ (radius length of the smaller circle)

$\therefore AB = AC$

(Q.E.D.)



5

[a] $\because ABCD$ is a cyclic quadrilateral

$\therefore m(\angle BAD)$
 $= 180^\circ - 70^\circ = 110^\circ$

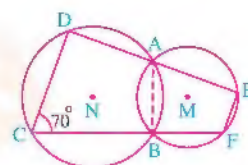
$\therefore ABFE$ is a cyclic quadrilateral and $\angle BAD$ is exterior of it.

$\therefore m(\angle EFB) = m(\angle BAD) = 110^\circ$ (First req.)

$\therefore m(\angle EFB) + m(\angle BCD) = 110^\circ + 70^\circ = 180^\circ$

and they are interior angles in the same side of \overleftrightarrow{FC}

$\therefore \overline{CD} \parallel \overline{EF}$ (Second req.)



[b] $\because \overline{AB}, \overline{AC}$

are tangent-segments to the circle

$\therefore AB = AC$

$\therefore m(\angle ACB) = \frac{180^\circ - 60^\circ}{2} = 60^\circ$ (1)

$\therefore m(\angle BEC)$ (inscribed)
 $= m(\angle ACB)$ (tangency) $= 60^\circ$ (2)

$\therefore EBCD$ is a cyclic quadrilateral

$\therefore m(\angle EBC) = 180^\circ - 120^\circ = 60^\circ$ (3)

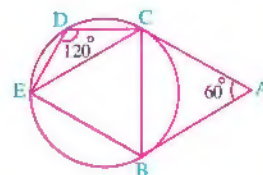
\therefore From (2), (3) in $\triangle EBC$:

$\therefore m(\angle BCE) = 60^\circ$

$\therefore \triangle BCE$ is equilateral. (Q.E.D. 1)

From (1), (3): $\therefore m(\angle ACB) = m(\angle EBC)$ and they are alternate angles

$\therefore \overline{AC} \parallel \overline{BE}$ (Q.E.D. 2)



Answers of model 2

1

- 1 d 2 a 3 b
4 d 5 c 6 b

2

[a] $\therefore m(\angle AMB) = 2m(\angle ADB)$
 $= 2 \times 70^\circ = 140^\circ$

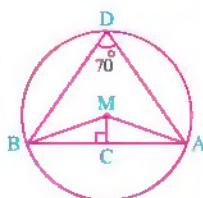
(central and inscribed angles subtended by \widehat{AB})

In $\triangle ABM$: $\therefore \overline{MC} \perp \overline{AB}$

$\therefore MA = MB = r$

$\therefore \overline{MC}$ bisects $\angle AMB$

$\therefore m(\angle AMC) = \frac{1}{2}m(\angle AMB) = \frac{1}{2} \times 140^\circ = 70^\circ$
(The req.)



[b] $\therefore M, N$ are two congruent circles

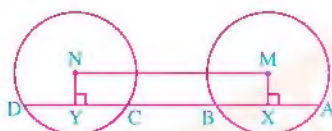
$\therefore AB = CD$

$\therefore \overline{MX} \perp \overline{AB}$

$\therefore \overline{NY} \perp \overline{CD}$

$\therefore MX = NY$, $\overline{MX} \parallel \overline{NY}$

\therefore MXYN is a rectangle. (Q.E.D.)



3

[a] $\therefore D$ is the midpoint of \overline{AB}

$\therefore \overline{MD} \perp \overline{AB}$

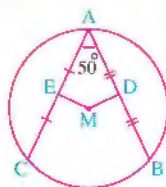
$\therefore m(\angle ADM) = 90^\circ$

$\therefore E$ is the midpoint of \overline{AC}

$\therefore \overline{ME} \perp \overline{AC}$ $\therefore m(\angle AEM) = 90^\circ$

From the quadrilateral ADME :

$\therefore m(\angle DME) = 360^\circ - (90^\circ + 90^\circ + 50^\circ) = 130^\circ$
(The req.)



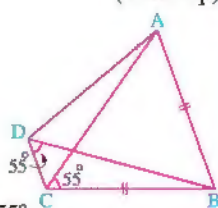
[b] In $\triangle ABC$:

$\therefore AB = BC$

$\therefore m(\angle BAC) = m(\angle ACB) = 55^\circ$

$\therefore m(\angle BDC) = m(\angle BAC) = 55^\circ$
and they are drawn on \overline{BC} and on one side of it

\therefore ABCD is a cyclic quadrilateral. (Q.E.D.)



4

[a] In $\triangle AMC$:

$\therefore AM = MC = r$

$\therefore m(\angle MAC) = m(\angle ACM)$

$\therefore m(\angle BAC) = m(\angle MAC)$

$\therefore m(\angle BAC) = m(\angle ACM)$
and they are alternate angles.

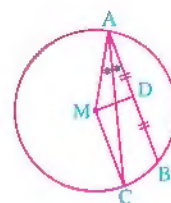
$\therefore \overline{AB} \parallel \overline{CM}$

$\therefore D$ is the midpoint of \overline{AB}

$\therefore \overline{MD} \perp \overline{AB}$

$\therefore \overline{DM} \perp \overline{CM}$

$\therefore \overline{AB} \parallel \overline{CM}$
(Q.E.D.)



[b] $\therefore \overline{AC}$ is a tangent to the circle M at A

$\therefore \overline{MA} \perp \overline{AC}$

$\therefore m(\angle CAM) = 90^\circ$

$\therefore \overline{BD}$ is a tangent to the circle M at B

$\therefore \overline{MB} \perp \overline{BD}$

$\therefore m(\angle EBM) = 90^\circ$

\therefore In $\triangle CAM, EBM$:

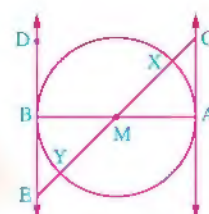
$$\begin{cases} m(\angle CAM) = m(\angle EBM) = 90^\circ \\ m(\angle AMC) = m(\angle BME) \text{ (V.O.A.)} \\ MA = MB \text{ (lengths of two radii)} \end{cases}$$

\therefore The two triangles are congruent and we deduce that $CM = EM$

$\therefore XM = YM$ (lengths of two radii)

\therefore by subtracting

$\therefore CX = YE$ (Q.E.D.)



5

[a] $\therefore \overline{XA}, \overline{XB}$

are two tangents to the circle

$\therefore XA = XB$

\therefore In $\triangle ABX$

$m(\angle XAB) = m(\angle XBA) = \frac{180^\circ - 50^\circ}{2} = 65^\circ$

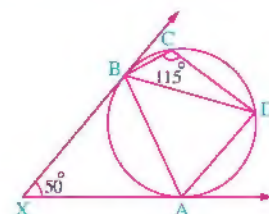
\therefore ABCD is a cyclic quadrilateral

$\therefore m(\angle BAD) + m(\angle DCB) = 180^\circ$

$\therefore m(\angle BAD) = 180^\circ - 115^\circ = 65^\circ$

$\therefore m(\angle XAB) = m(\angle BAD)$

$\therefore \overline{AB}$ bisects $\angle DAX$ (Q.E.D.1)



$$\begin{aligned} \therefore m(\angle ADB) & \text{ (inscribed)} \\ & = m(\angle XAB) \text{ (tangency)} = 65^\circ \end{aligned}$$

$$\therefore m(\angle BAD) = m(\angle ADB)$$

$$\therefore \text{In } \triangle ABD : BD = BA \quad (\text{Q.E.D.2})$$

$$[b] \therefore AB = CD$$

$$\therefore m(\widehat{AB}) = m(\widehat{CD})$$

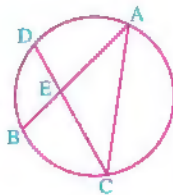
Subtracting $m(\widehat{BD})$ from both sides

$$\therefore m(\widehat{AD}) = m(\widehat{BC})$$

$$\therefore m(\angle ACD) = m(\angle BAC)$$

$$\therefore \text{In } \triangle ACE : AE = CE$$

$$\therefore \triangle ACE \text{ is an isosceles triangle.} \quad (\text{Q.E.D.})$$



Answers of model 3

1

1 a

2 a

3 b

4 d

5 c

6 c

2

$$[a] \therefore \overline{MN} \text{ is the line of centres}$$

$$\therefore \overline{AB} \text{ is the common chord.}$$

$$\therefore \overline{AB} \perp \overline{MN}$$

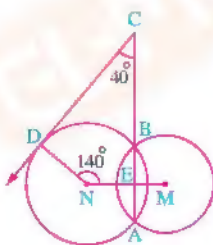
$$\therefore m(\angle BEN) = 90^\circ$$

In the quadrilateral CDNE :

$$\therefore m(\angle CDN) = 360^\circ - (140^\circ + 40^\circ + 90^\circ) = 90^\circ$$

$$\therefore \overline{ND} \perp \overline{CD}$$

$$\therefore \overline{CD} \text{ is a tangent to the circle N at D} \quad (\text{Q.E.D.})$$



$$[b] \therefore AB = CD$$

(properties of the rectangle)

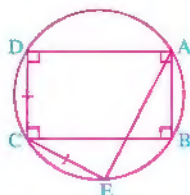
$$\therefore CE = CD$$

$$\therefore AB = CE$$

$$\therefore m(\widehat{AB}) = m(\widehat{CE}) \text{ and adding } m(\widehat{BE}) \text{ to both sides.}$$

$$\therefore m(\widehat{AE}) = m(\widehat{BC})$$

$$\therefore AE = BC \quad (\text{Q.E.D.})$$



3

$$[a] \text{ State by yourself.}$$

$$\begin{aligned} [b] \therefore m(\widehat{BC}) & = 2m(\angle A) \\ & = 2 \times 30^\circ = 60^\circ \end{aligned}$$

$$\begin{aligned} \therefore m(\angle E) & = \frac{1}{2} [m(\widehat{AD}) - m(\widehat{BC})] \\ \therefore 50^\circ & = \frac{1}{2} [m(\widehat{AD}) - 60^\circ] \end{aligned}$$

$$\therefore 100^\circ = m(\widehat{AD}) - 60^\circ$$

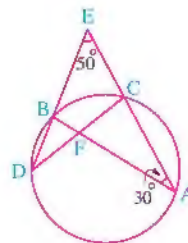
$$\therefore m(\widehat{AD}) = 160^\circ$$

(First req.)

$$\therefore m(\angle AFD) = \frac{1}{2} [m(\widehat{AD}) + m(\widehat{BC})]$$

$$\therefore m(\angle AFD) = \frac{1}{2} [160^\circ + 60^\circ] = 110^\circ$$

(Second req.)



4

$$\begin{aligned} [a] \therefore m(\angle ACB) & = \frac{1}{2} m(\angle AMB) = 60^\circ \\ & \text{(inscribed and central angles} \\ & \text{subtended the same arc } \widehat{AB}) \quad (1) \end{aligned}$$

$$\therefore \overline{CD} \parallel \overline{AB}$$

$$\therefore m(\widehat{AC}) = m(\widehat{BC})$$

$$\therefore AC = BC$$

(2)

From (1) and (2) :

$$\therefore \triangle CAB \text{ is equilateral.} \quad (\text{Q.E.D.})$$

$$[b] \therefore \overline{AB} \parallel \overline{DC}, \overline{AD}$$

is a transversal to them.

$$\begin{aligned} \therefore m(\angle A) + m(\angle D) & = 180^\circ \quad (1) \end{aligned}$$

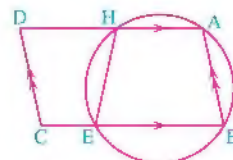
but $\angle CEH$ is an exterior angle of the cyclic quadrilateral ABEH

$$\therefore m(\angle CEH) = m(\angle A) \quad (2)$$

From (1) and (2) :

$$\therefore m(\angle CEH) + m(\angle D) = 180^\circ$$

$$\therefore HDCE \text{ is a cyclic quadrilateral.} \quad (\text{Q.E.D.})$$



5

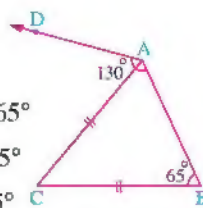
$$[a] \text{ In } \triangle ABC :$$

$$\therefore AC = BC$$

$$\therefore m(\angle BAC) = m(\angle ABC) = 65^\circ$$

$$\therefore m(\angle CAD) = 130^\circ - 65^\circ = 65^\circ$$

$$\therefore m(\angle B) = m(\angle CAD) = 65^\circ$$



$\therefore \overrightarrow{AD}$ is a tangent to the circle passing through the vertices of the triangle ABC (Q.E.D.)

[b] $\therefore MF = ME$
(lengths of two radii)

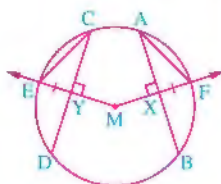
$\therefore XF = YE$

$\therefore MX = MY$

$\therefore \overline{MX} \perp \overline{AB}, \overline{MY} \perp \overline{CD}$

$\therefore AB = CD$

(Q.E.D.1)



$\therefore \overline{MX} \perp \overline{AB}$

$\therefore X$ is the midpoint of \overline{AB}

$\therefore AX = \frac{1}{2} AB$, $\therefore \overline{MY} \perp \overline{CD}$

$\therefore Y$ is the midpoint of \overline{CD}

$\therefore CY = \frac{1}{2} CD$, $\therefore AB = CD$

$\therefore AX = CY$

\therefore In $\Delta \Delta AXF, CYE$

$$\begin{cases} AX = CY \\ XF = YE \\ m(\angle AXF) = m(\angle CYE) = 90^\circ \end{cases}$$

$\therefore \Delta AXF \cong \Delta CYE$ $\therefore AF = CE$ (Q.E.D.2)

Model Examinations of the School Book



on Geometry

Model

1

Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

1 The inscribed angle drawn in a semicircle is

- (a) an acute. (b) an obtuse. (c) a straight. (d) a right.

2 In the opposite figure :

Circle of centre M

If $m(\widehat{AB}) = 50^\circ$, then $m(\angle ADB) = \dots\dots\dots$

- (a) 25° (b) 50° (c) 100° (d) 150°

3 The number of symmetric axes of any circle is

- (a) zero (b) 1 (c) 2 (d) an infinite number.

4 In the opposite figure :

If $m(\angle A) = 120^\circ$, then $m(\angle C) = \dots\dots\dots$

- (a) 60° (b) 90°
(c) 120° (d) 180°

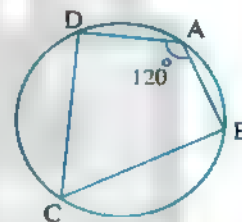
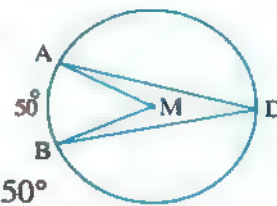
5 If the straight line L is a tangent to the circle M of diameter length 8 cm.

, then the distance between L and the centre of the circle equals cm.

- (a) 3 (b) 4 (c) 6 (d) 8

6 The surface of the circle M \cap the surface of the circle N = {A} and the radius length of one of them is 3 cm. and $MN = 8$ cm. , then the radius length of the other circle equals cm.

- (a) 5 (b) 6 (c) 11 (d) 16



2 [a] Complete and prove that :

In a cyclic quadrilateral , each two opposite angles are

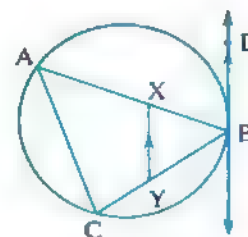
[b] In the opposite figure :

ABC is a triangle inscribed in a circle

, \overrightarrow{BD} is a tangent to the circle at B

, $X \in \overline{AB}$, $Y \in \overline{BC}$ where $\overline{XY} \parallel \overline{BD}$

Prove that : AXYC is a cyclic quadrilateral.

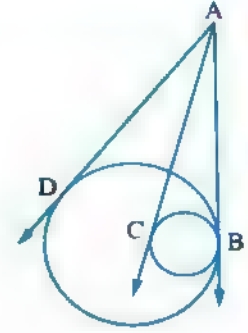


Geometry

3 [a] In the opposite figure :

- Two circles are touching internally at B
 \overrightarrow{AB} is a common tangent
 \overrightarrow{AC} is a tangent to the smaller circle at C
 \overrightarrow{AD} is a tangent to the greater circle at D
 $AC = 15$ cm. , $AB = (2x - 3)$ cm.
 and $AD = (y - 2)$ cm.

Find : The value of each of x and y

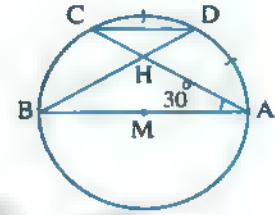


[b] In the opposite figure :

- \overline{AB} is a diameter in the circle M
 $C \in$ the circle M , $m(\angle CAB) = 30^\circ$
 D is midpoint of \widehat{AC} , $\overline{DB} \cap \overline{AC} = \{H\}$

1 Find : $m(\angle BDC)$ and $m(\widehat{AD})$

2 Prove that : $\overline{AB} \parallel \overline{DC}$

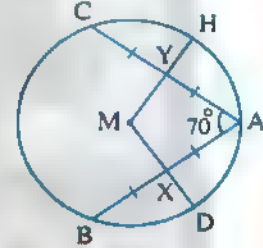


4 [a] In the opposite figure :

- \overline{AB} and \overline{AC} are two chords equal in length in circle M
 X is the midpoint of \overline{AB} , Y is the midpoint of \overline{AC}
 $m(\angle CAB) = 70^\circ$

1 Calculate : $m(\angle DMH)$

2 Prove that : $XD = YH$

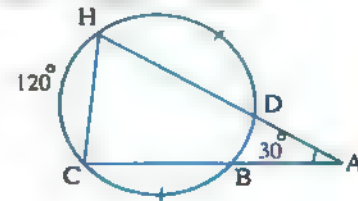


[b] In the opposite figure :

- $m(\angle A) = 30^\circ$, $m(\widehat{HC}) = 120^\circ$
 $m(\widehat{BC}) = m(\widehat{DH})$

1 Find : $m(\widehat{BD})$ the minor

2 Prove that : $AB = AD$

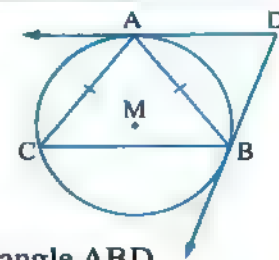


5 [a] In the opposite figure :

- \overline{DA} and \overline{DB} are two tangents of the circle M
 and $AB = AC$

Prove that :

\overline{AC} is a tangent to the circle passing through the vertices of the triangle ABD

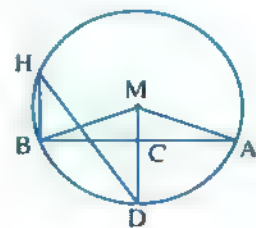


[b] In the opposite figure :

C is the midpoint of \overline{AB} , $\overline{MC} \cap$ the circle $M = \{D\}$

, $m(\angle MAB) = 20^\circ$

Find : $m(\angle BHD)$ and $m(\widehat{ADB})$



Model 2

1 Choose the correct answer from those given :

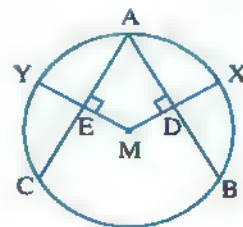
- 1 The measure of the arc which equals half the measure of the circle equals
 (a) 360° (b) 180° (c) 120° (d) 90°
- 2 The number of common tangents of two touching circles externally equals
 (a) 0 (b) 1 (c) 2 (d) 3
- 3 The measure of the inscribed angle drawn in a semicircle equals
 (a) 45° (b) 90° (c) 120° (d) 80°
- 4 The angle of tangency is included between
 (a) two chords. (b) two tangents.
 (c) a chord and a tangent. (d) a chord and a diameter.
- 5 ABCD is a cyclic quadrilateral, $m(\angle A) = 60^\circ$, then $m(\angle C) =$
 (a) 60° (b) 30° (c) 90° (d) 120°
- 6 If M, N are two touching circles internally, their radii lengths are 5 cm, 9 cm.
 , then $MN =$ cm.
 (a) 14 (b) 4 (c) 5 (d) 9

2 [a] In the opposite figure :

$AB = AC$, $\overline{MD} \perp \overline{AB}$,

$\overline{ME} \perp \overline{AC}$

Prove that : $XD = YE$



Geometry

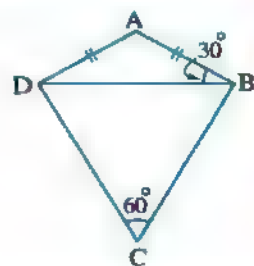
[b] In the opposite figure :

ABCD is a quadrilateral in which $AB = AD$,

$m(\angle ABD) = 30^\circ$,

$m(\angle C) = 60^\circ$

Prove that : ABCD is a cyclic quadrilateral.



3 [a] State two cases of a cyclic quadrilateral.

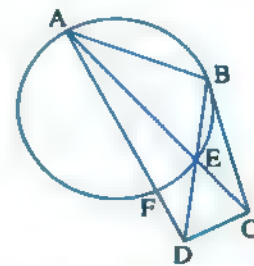
[b] In the opposite figure :

\overline{BC} is a tangent at B ,

E is the midpoint of \widehat{BF}

Prove that :

ABCD is a cyclic quadrilateral.



4 [a] In the opposite figure :

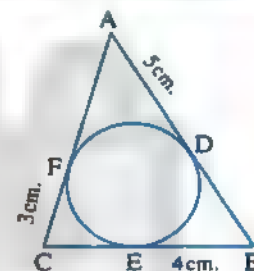
A circle is drawn touches the sides of a triangle

ABC , \overline{AB} , \overline{BC} , \overline{AC} at

D , E , F , $AD = 5$ cm ,

$BE = 4$ cm. , $CF = 3$ cm.

Find the perimeter of $\triangle ABC$

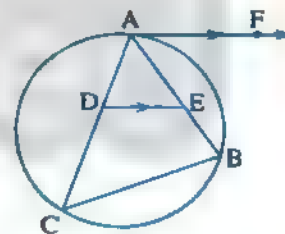


[b] In the opposite figure :

\overline{AF} is a tangent to the circle at A , $\overline{AF} \parallel \overline{DE}$

Prove that :

DEBC is a cyclic quadrilateral.



5 In the opposite figure :

\overline{AB} , \overline{AC} are two tangents

to the circle at B , C

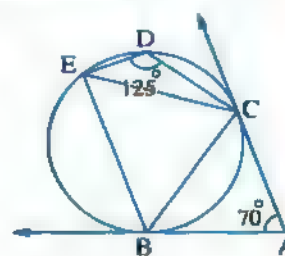
, $m(\angle A) = 70^\circ$,

$m(\angle CDE) = 125^\circ$

Prove that :

1 $CB = CE$

2 $\overline{AC} \parallel \overline{BE}$

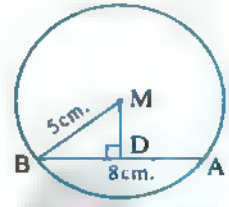


Model examination for the merge students

Answer the following questions in the same paper : (Calculator is allowed)

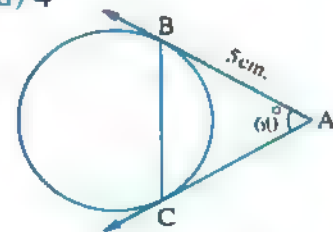
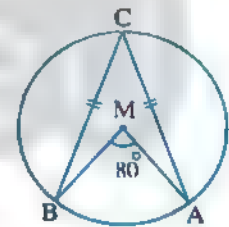
1 Complete each of the following :

- 1 The longest chord in the circle is called
- 2 The straight line passing through the center of the circle and the midpoint of any chord is
- 3 The two tangent-segments drawn to a circle from a point outside it are in length.
- 4 In the opposite figure :
The length of \overline{MD} = cm.
- 5 The number of symmetry axes of a circle is
- 6 If \overline{AC} is a diameter in a circle M , then $m(\widehat{AC}) = \dots\dots\dots^\circ$



2 Choose the correct answer from those given :

- 1 If A \in the circle M of diameter length 6 cm.
then $MA = \dots\dots\dots$ cm.
(a) 3 (b) 4
(c) 5 (d) 6
- 2 In the opposite figure :
 $m(\angle ACB) = \dots\dots\dots$
(a) 40° (b) 80°
(c) 90° (d) 180°
- 3 The number of the common tangents of two distant circles is
(a) 1 (b) 2 (c) 3 (d) 4
- 4 In the opposite figure :
The length of $\overline{BC} = \dots\dots\dots$ cm.
(a) 3 (b) 4
(c) 5 (d) 6
- 5 The number of circles which can be drawn passing through the endpoints of a line segment \overline{AB} equals
(a) 1 (b) 2 (c) 3 (d) an infinite number.



Geometry

6 In the opposite figure :

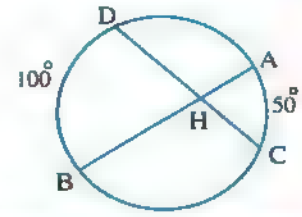
$$m(\angle AHC) = \dots\dots\dots$$

(a) 25°

(b) 50°

(c) 75°

(d) 100°



3 Put (✓) for the correct statement , (X) for the incorrect statement :

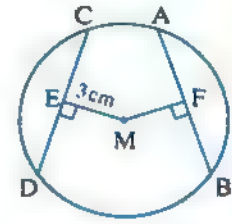
- 1 If M , N are two touching externally circles with radii lengths are $r_1 = 5$ cm. , $r_2 = 3$ cm. , then $MN = 15$ cm. ()

2 In the opposite figure :

If $AB = CD$,

$ME = 3$ cm. , then

$MF = 3$ cm. ()

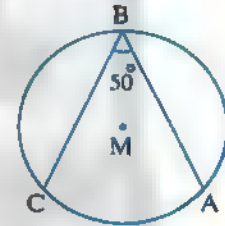


3 The quadrilateral ABCD is a cyclic quadrilateral if

$m(\angle A) + m(\angle C) = 90^\circ$ ()

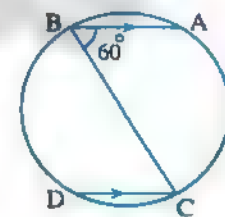
4 In the opposite figure :

$m(\widehat{AC}) = 100^\circ$ ()



5 In the opposite figure :

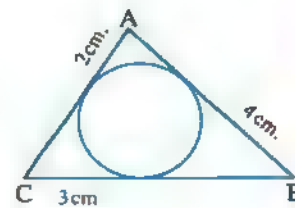
$m(\widehat{AB}) + m(\widehat{CD}) = 300^\circ$ ()



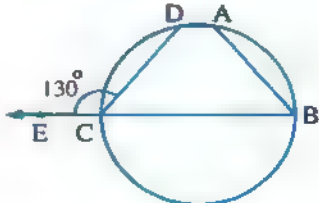
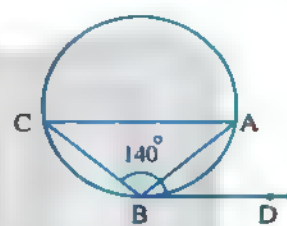
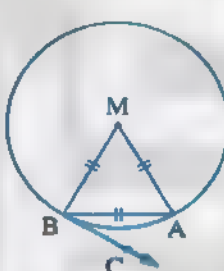
6 In the opposite figure :

The perimeter of

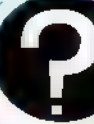
$\Delta ABC = 9$ cm. ()



4 Join from the column (A) to the suitable one of the column (B) :

(A)	(B)
<p>1 The measure of the inscribed angle which is drawn in a semicircle equals</p>	• 130°
<p>2 In the opposite figure : $m(\angle A) = \dots\dots\dots$</p> 	• 40°
<p>3 In the opposite figure : \overline{BD} is a tangent at B , $m(\angle DBC) = 140^\circ$, then $m(\angle A) = \dots\dots\dots$</p> 	• 90°
<p>4 The radius of the circumcircle of the vertices of right-angled triangle of hypotenuse length 10 cm. equals cm.</p>	• 30°
<p>5 In the opposite figure : $\triangle MAB$ is an equilateral triangle , \overline{BC} is a tangent at B , , then $m(\angle ABC) = \dots\dots\dots$</p> 	• 2 : 1
<p>6 The ratio between the measures of the central angle and inscribed angle subtended by the same arc is</p>	• 5

Governorates Examinations



on Geometry

1

Cairo Governorate



Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

- 1 The area of the rhombus with diagonal lengths 6 cm. , 8 cm. is cm²
 (a) 2 (b) 14 (c) 24 (d) 48
- 2 Two distant circles M and N with radii lengths 6 cm and 8 cm respectively , then MN 14 cm.
 (a) < (b) > (c) = (d) ≥
- 3 The measure of the inscribed angle is the measure of the central angle subtended by the same arc.
 (a) half (b) twice (c) quarter (d) third
- 4 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals the length of the hypotenuse.
 (a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{2}$ (d) 2
- 5 In the cyclic quad. ABCD , if $m(\angle A) = \frac{1}{2} m(\angle C)$, then $m(\angle A) = \dots\dots\dots^\circ$
 (a) 20 (b) 30 (c) 60 (d) 120
- 6 The angle of measure 40° is the complemented angle of the angle of measure °
 (a) 320 (b) 140 (c) 60 (d) 50

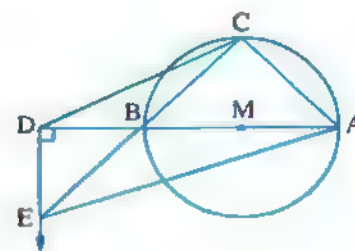
2 [a] Mention two cases of the cyclic quadrilateral.

[b] In the opposite figure :

\overline{AB} is a diameter of the circle M , $D \in \overline{AB}$
 $D \notin \overline{AB}$, $\overline{DE} \perp \overline{AB}$, $C \in \widehat{AB}$
 $\overline{CB} \cap \overline{DE} = \{E\}$

1 Find : $m(\angle ACB)$

2 Prove that : The figure ACDE is a cyclic quadrilateral.



- 3 [a] Find the measure of the arc which represents $\frac{1}{3}$ of the measure of the circle.

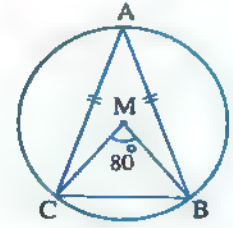
[b] In the opposite figure :

$\triangle ABC$ is drawn inside the circle M

, $AB = AC$, $m(\angle BMC) = 80^\circ$

Find : 1 $m(\angle ABC)$

2 The measure of the major arc \widehat{BC}



- 4 [a] In the opposite figure :

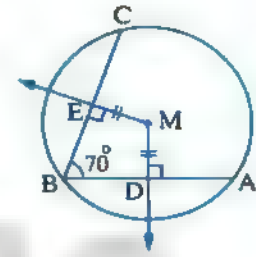
\overline{AB} and \overline{BC} are two chords in the circle M , $\overline{MD} \perp \overline{AB}$

, $\overline{ME} \perp \overline{CB}$, $MD = ME$

, $m(\angle ABC) = 70^\circ$

1 Find : $m(\angle DME)$

2 Prove that : $AB = CB$



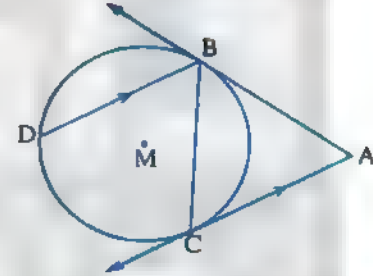
[b] In the opposite figure :

\overline{AB} and \overline{AC} are two tangents to

the circle M at B and C respectively

, $\overline{BD} \parallel \overline{AC}$

Prove that : \overline{BC} bisects $\angle ABD$



- 5 [a] Using the geometric tools , draw \overline{AB} with length 6 cm , and then draw a circle passing through the two points A , B with radius length 4 cm. What is the length of the radius of the smallest circle passing through the two points A and B ?

[b] In the opposite figure :

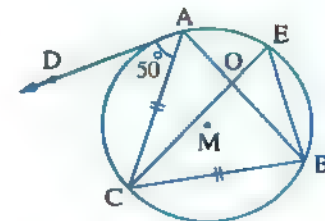
A circle M , $AC = BC$

, \overline{AD} is a tangent to the circle at A , $m(\angle CAD) = 50^\circ$

1 Find : $m(\angle ABC)$, $m(\angle BEC)$

2 Prove that :

\overline{BC} is a tangent to the circle passing through the vertices of the triangle BEO



Geometry

2

Giza Governorate



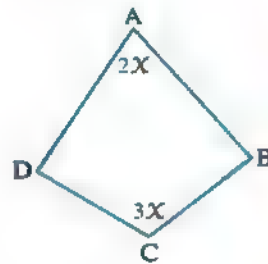
Answer the following questions :

1 Choose the correct answer :

1 In the opposite figure :

ABCD is a cyclic quadrilateral
 $m(\angle A) = 2X$, $m(\angle C) = 3X$
 then the value of $X = \dots\dots\dots^\circ$

- (a) 20 (b) 30
 (c) 32 (d) 36



2 If the ratio between the perimeters of two squares is 1 : 2 , then the ratio between their areas equals

- (a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1

3 The measure of the inscribed angle in a semicircle equals

- (a) 45 (b) 90 (c) 120 (d) 180

4 The median of the triangle divides its surface into two triangles

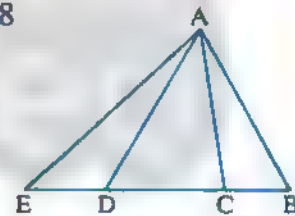
- (a) congruent. (b) equal in area. (c) isosceles. (d) right-angled.

5 If the two circles M , N are touching internally , their radii lengths are 3 cm. , 5 cm. , then MN = cm.

- (a) 3 (b) 5 (c) 2 (d) 8

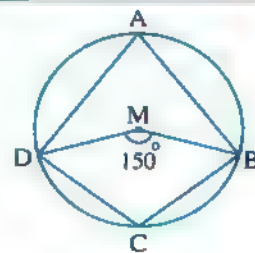
6 The number of triangles in the opposite figure equals

- (a) 3 (b) 4
 (c) 5 (d) 6



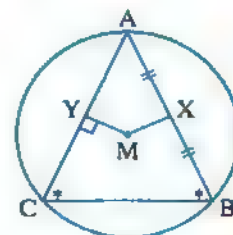
2 [a] In the opposite figure :

A circle of centre M

 $m(\angle BMD) = 150^\circ$ Find with proof : $m(\angle C)$ 

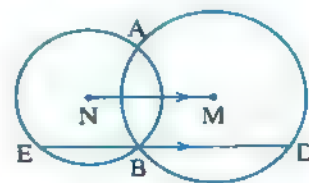
[b] In the opposite figure :

ABC is an inscribed triangle in a circle M

in which $m(\angle B) = m(\angle C)$, X is the midpoint of \overline{AB} , $MY \perp \overline{AC}$ Prove that : $MX = MY$ 

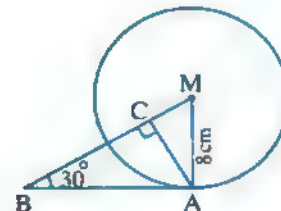
3 [a] In the opposite figure :

M, N are two intersecting circles at A, B
 $\overleftrightarrow{BD} \parallel \overleftrightarrow{MN}$ and intersects the two circles at D, E
 Prove that : $DE = 2 MN$



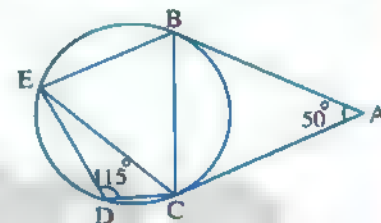
[b] In the opposite figure :

\overleftrightarrow{AB} is a tangent to the circle M at A
 $MA = 8 \text{ cm}$, $m(\angle ABM) = 30^\circ$, $\overleftrightarrow{AC} \perp \overleftrightarrow{MB}$
 Find : The length of each of \overleftrightarrow{AB} , \overleftrightarrow{AC}



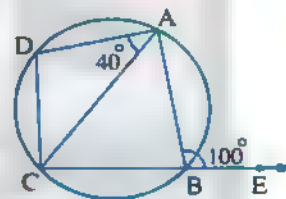
4 [a] In the opposite figure :

\overleftrightarrow{AB} , \overleftrightarrow{AC} are two tangent-segments to the circle at B, C
 $m(\angle A) = 50^\circ$, $m(\angle CDE) = 115^\circ$
 Prove that : 1) \overleftrightarrow{BC} bisects $\angle ABE$
 2) $CB = CE$



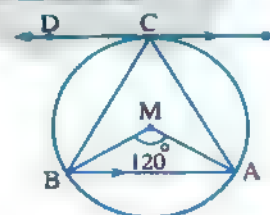
[b] In the opposite figure :

$m(\angle ABE) = 100^\circ$
 $m(\angle CAD) = 40^\circ$
 Prove that : $m(\widehat{CD}) = m(\widehat{AD})$



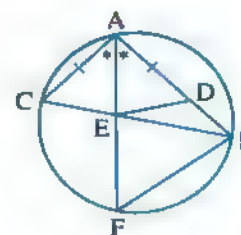
5 [a] In the opposite figure :

\overleftrightarrow{CD} is a tangent to the circle M at C
 $\overleftrightarrow{CD} \parallel \overleftrightarrow{AB}$
 $m(\angle AMB) = 120^\circ$
 Prove that : $\triangle CAB$ is an equilateral triangle.



[b] In the opposite figure :

$AC = AD$, \overleftrightarrow{AE} bisects $\angle BAC$
 and cuts \overleftrightarrow{BC} at E and the circle at F
 Prove that : $BDEF$ is a cyclic quadrilateral.



Geometry

3 Alexandria Governorate



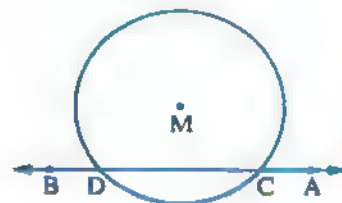
Answer the following questions : (Calculators are permitted)

1 Choose the correct answer from those given :

1 In the opposite figure :

$\overline{AB} \cap$ the surface of the circle M =

- (a) $\{C, D\}$ (b) \overline{CD}
(c) \overline{CD} (d) \emptyset



2 $\angle A$ and $\angle B$ are two complementary angles , $\angle B$ and $\angle C$ are two supplementary angles , $m(\angle A) = 30^\circ$, then $m(\angle C) = \dots^\circ$

- (a) 30 (b) 60 (c) 90 (d) 120

3 If the surface of the circle M \cap the surface of the circle N = $\{A\}$ and the radius length of one of them equals 3 cm and $MN = 8$ cm. , then the radius length of the other circle equals cm.

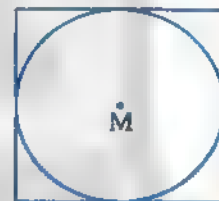
- (a) 5 (b) 6 (c) 11 (d) 16

4 In the opposite figure :

If the side length of the square = 10 cm.

, then the surface area of the circle = cm^2

- (a) 100π (b) 25π
(c) 50π (d) 40π



5 A circle can be drawn passing through the vertices of a

- (a) rhombus (b) parallelogram (c) trapezium (d) rectangle

6 The rhombus whose two diagonal lengths are 12 cm. and 16 cm. , then its side length equals cm.

- (a) 6 (b) 8 (c) 10 (d) 20

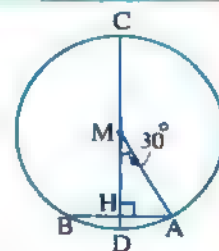
2 [a] In the opposite figure :

\overline{CD} is a diameter in the circle M

, $AB = 10$ cm. , $\overline{MH} \perp \overline{AB}$

, $m(\angle AMD) = 30^\circ$

Find : The length of \overline{CD}



[b] ABCD is a quadrilateral inscribed in a circle , E is a point outside the circle , \overline{EA} and \overline{EB} are two tangents to the circle at A and B , if $m(\angle AEB) = 70^\circ$ and $m(\angle ADC) = 125^\circ$, prove that : $AB = AC$

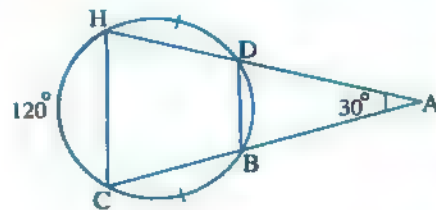
3 [a] In the opposite figure :

$$m(\angle A) = 30^\circ, m(\widehat{HC}) = 120^\circ$$

$$, m(\widehat{BC}) = m(\widehat{DH})$$

1 Find : $m(\widehat{BD})$ «the minor arc»

2 Prove that : $AB = AD$



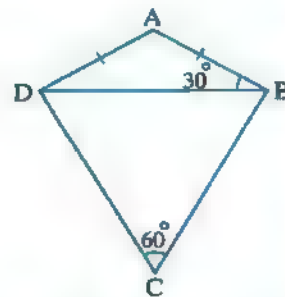
[b] In the opposite figure :

ABCD is a quadrilateral , $AB = AD$

$$, m(\angle ABD) = 30^\circ$$

$$, m(\angle C) = 60^\circ$$

Prove that : ABCD is a cyclic quadrilateral.

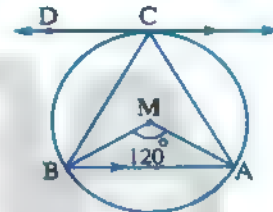


4 [a] In the opposite figure :

\overrightarrow{CD} is a tangent to the circle at C

$$, \overrightarrow{CD} \parallel \overrightarrow{AB}, m(\angle AMB) = 120^\circ$$

Prove that : The triangle CAB is an equilateral triangle.



[b] In the opposite figure :

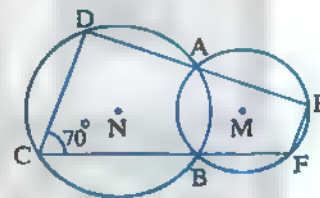
M and N are two intersecting circles at A and B

\overrightarrow{AD} is drawn to intersect the circle M at E and the circle N at D

\overrightarrow{BC} is drawn to intersect the circle M at F and the circle N at C

$$, m(\angle C) = 70^\circ$$

Prove that : $\overrightarrow{CD} \parallel \overrightarrow{EF}$

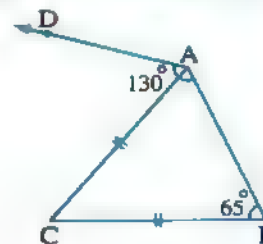


5 [a] In the opposite figure :

$$AC = BC, m(\angle ABC) = 65^\circ$$

$$, m(\angle DAB) = 130^\circ$$

Prove that : \overrightarrow{AD} is a tangent to the circle passing through the vertices of the triangle ABC



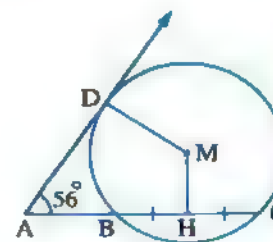
[b] In the opposite figure :

\overrightarrow{AD} is a tangent to the circle M

\overrightarrow{AC} intersects the circle M at B , C

$$, m(\angle A) = 56^\circ \text{ and H is the midpoint of } \overline{BC}$$

Find with proof : $m(\angle DMH)$



Geometry

4 El-Kalyoubia Governorate



Answer the following questions :

1 Choose the correct answer :

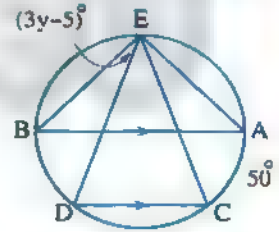
- 1 ABC is a triangle in which : $(AB)^2 > (BC)^2 + (AC)^2$, then $\angle C$ is
 (a) acute. (b) right. (c) obtuse. (d) straight.
- 2 If M and N are two intersecting circles whose radii length are 5 cm and 2 cm, then $MN \in$
 (a) $]3, 7[$ (b) $[3, 7[$ (c) $]3, 7]$ (d) $[3, 7]$
- 3 If $\triangle ABC \sim \triangle XYZ$, $m(\angle A) = 50^\circ$, $m(\angle B) = 60^\circ$, then $m(\angle Z) =$
 (a) 90 (b) 110 (c) 10 (d) 70
- 4 The measure of the central angle which is opposite to an arc of length $\frac{1}{3} \pi r$ equals
 (a) 30 (b) 60 (c) 120 (d) 240
- 5 ABC is a right-angled triangle at B, $\overline{BD} \perp \overline{AC}$ where $\overline{BD} \cap \overline{AC} = \{D\}$, then the projection of \overline{BD} on \overline{AC} is
 (a) A (b) B (c) C (d) D
- 6 If ABCD is a cyclic quadrilateral, then $m(\angle BAC) = m(\angle \dots)$
 (a) BCA (b) DBA (c) BDC (d) ACD

2 [a] In the opposite figure :

$\overline{AB} \parallel \overline{CD}$, $m(\widehat{AC}) = 50^\circ$

, $m(\angle BED) = (3y - 5)^\circ$

Find : The value of y



- [b] Using your geometric tools, draw \overline{AB} with length 4 cm, then draw a circle passing through the two points A and B whose diameter length is 5 cm.
 How many circles can be drawn ? (Don't erase the arcs).

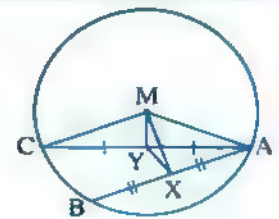
3 [a] In the opposite figure :

A circle with centre M

, X and Y are the midpoints of \overline{AB} and \overline{AC} respectively.

Prove that : 1 AXYM is a cyclic quadrilateral.

2 $m(\angle MXY) = m(\angle MCY)$



[b] In the opposite figure :

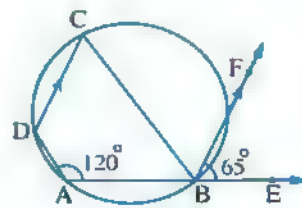
$$m(\angle A) = 120^\circ, m(\angle EBF) = 65^\circ$$

$$\overline{DC} \parallel \overline{BF}$$

Find with proof :

1 $m(\angle C)$

2 $m(\angle D)$



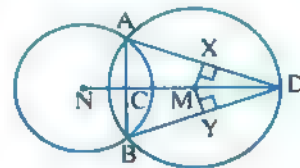
4 [a] In the opposite figure :

$$\text{Circle } M \cap \text{circle } N = \{A, B\}$$

$$\overline{AB} \cap \overline{MN} = \{C\}, D \in \overline{MN}$$

$$\overline{MX} \perp \overline{AD}, \overline{MY} \perp \overline{BD}$$

Prove that : $MX = MY$



[b] ABC is a triangle inscribed in a circle, \overline{AD} is a tangent to the circle at A

$$X \in \overline{AB}, Y \in \overline{AC}, \text{ where } \overline{XY} \parallel \overline{BC}$$

Prove that : \overline{AD} is a tangent to the circle passing through the points A, X and Y

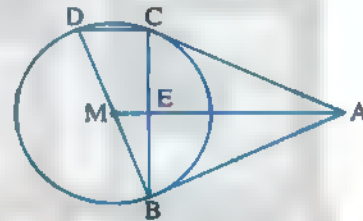
5 [a] In the opposite figure :

\overline{AB} and \overline{AC} are two tangent-segments to the circle M

$$\overline{AM} \cap \overline{CB} = \{E\}$$

and \overline{BD} is a diameter of the circle.

Prove that : $\overline{AM} \parallel \overline{CD}$



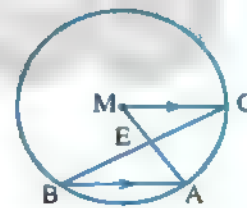
[b] In the opposite figure :

\overline{AB} is a chord in the circle M

$$\overline{CM} \parallel \overline{AB}$$

$$\overline{BC} \cap \overline{AM} = \{E\}$$

Prove that : $BE > AE$



5

El-Sharkia Governorate



Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from the given ones :

1 A circle can be drawn passing through the vertices of a

(a) rhombus.

(b) rectangle.

(c) trapezium.

(d) parallelogram.

Geometry

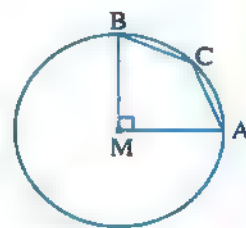
- 2 A circle with diameter length 10 cm. , the straight line L is distant from its centre by 5 cm. , then the straight line L is
- (a) a tangent. (b) a secant.
(c) outside the circle. (d) a diameter of the circle.
- 3 The number of common tangents of two touching circles externally equals
- (a) zero (b) 1 (c) 2 (d) 3
- 4 If M , N are two touching circles externally , the lengths of their radii are 2 cm. , 4 cm. respectively , then the area of the circle with diameter \overline{MN} equals cm^2
- (a) 36π (b) 9π (c) 16π (d) 4π

- 5 In the opposite figure :

A circle M , $\overline{MA} \perp \overline{MB}$

, then $m(\angle ACB) = \dots\dots\dots$

- (a) 45° (b) 90°
(c) 145° (d) 135°

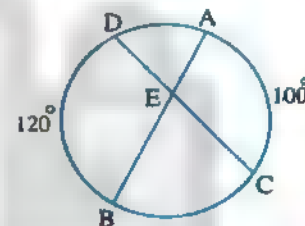


- 6 In the opposite figure :

$m(\widehat{AC}) = 100^\circ$, $m(\widehat{DB}) = 120^\circ$

, then $m(\angle AEC) = \dots\dots\dots$

- (a) 110° (b) 55°
(c) 70° (d) 100°



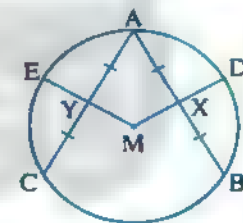
- 2 [a] In the opposite figure :

\overline{AB} and \overline{AC} are two equal chords in circle M

, X is the midpoint of \overline{AB}

, Y is the midpoint of \overline{AC}

Prove that : $XD = YE$

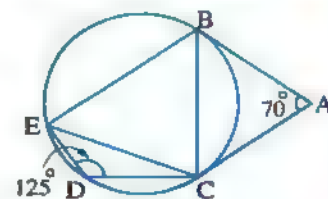


- [b] In the opposite figure :

\overline{AB} and \overline{AC} are two tangent-segments to the circle at B and C

, $m(\angle A) = 70^\circ$, $m(\angle CDE) = 125^\circ$

Prove that : \overline{BC} bisects $\angle ABE$



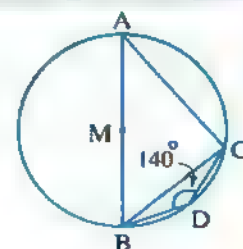
- 3 [a] In the opposite figure :

\overline{AB} is a diameter in the circle M

, $m(\widehat{BD}) = m(\widehat{DC})$, $m(\angle BDC) = 140^\circ$

Find with proof : 1 $m(\angle ABC)$

2 $m(\angle ABD)$



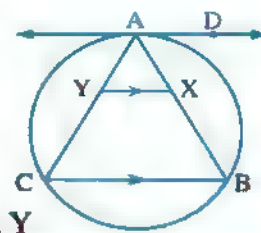
[b] In the opposite figure :

\overrightarrow{AD} is a tangent to the circle at A , $X \in \overline{AB}$

, $Y \in \overline{AC}$ and $\overline{XY} \parallel \overline{BC}$

Prove that :

\overrightarrow{AD} is a tangent to the circle which passes through the points A , X and Y

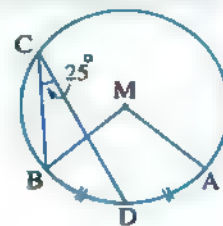


4 [a] In the opposite figure :

A circle M , D is the midpoint of \widehat{AB}

, $m(\angle DCB) = 25^\circ$

Find : $m(\angle AMB)$



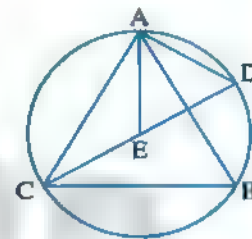
[b] In the opposite figure :

ABC is an equilateral triangle drawn in the circle

, $D \in \widehat{AB}$, $E \in \widehat{DC}$, where $AD = DE$

Prove that : 1 $\triangle ADE$ is an equilateral triangle.

2 $m(\angle DAB) = m(\angle EAC)$



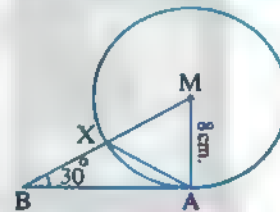
5 [a] In the opposite figure :

\overline{AB} is a tangent-segment to the circle M at A

, $AM = 8 \text{ cm}$, $m(\angle ABM) = 30^\circ$

1 Find : The length of \overline{AB}

2 Prove that : $\triangle XAB$ is an isosceles triangle.

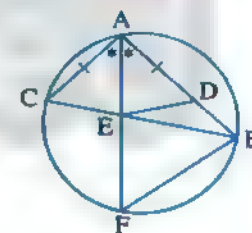


[b] In the opposite figure :

$AD = AC$

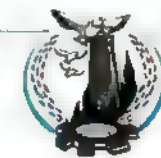
, \overrightarrow{AF} bisects $\angle BAC$

Prove that : DBFE is a cyclic quadrilateral.



6

El-Monofia Governorate



Answer the following questions : (Calculators are permitted)

1 Choose the correct answer from those given :

1 The axis of symmetry of a circle is

(a) the diameter.

(b) the chord.

(c) the straight line passing through the center.

(d) the tangent.

Geometry

- 2 XYZ is a triangle. If $(XY)^2 - (YZ)^2 > (XZ)^2$, then $\angle Y$ is
- (a) acute. (b) right. (c) obtuse. (d) reflex.

- 3 In the opposite figure :

If $AB = AC$, $BC = BD = AD$
 , then $m(\angle A) = \dots\dots\dots^\circ$

- (a) 30 (b) 36
 (c) 45 (d) 72

- 4 ABCD is a cyclic quadrilateral in which $m(\angle A) = 2m(\angle C)$
 , then $m(\angle C) = \dots\dots\dots^\circ$

- (a) 30 (b) 60 (c) 90 (d) 120

- 5 In the opposite figure :

A circle M, $MC = 4$ cm.

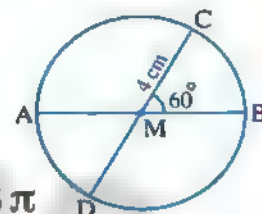
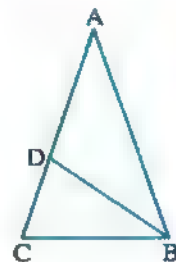
, $m(\angle CMB) = 60^\circ$

, then the length of $\widehat{BD} = \dots\dots\dots$ cm.

- (a) 4π (b) 8π (c) $\frac{8}{3}\pi$ (d) 16π

- 6 If $Y \in \overline{XZ}$ and $XY = 2YZ$, then the area of the square drawn on $\overline{XY} = \dots\dots\dots$
 The area of the square drawn on \overline{XZ}

- (a) $\frac{9}{4}$ (b) $\frac{4}{9}$ (c) 2 (d) $\frac{1}{2}$

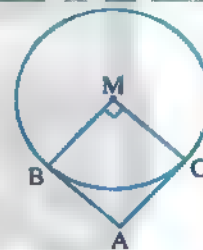


- 2 [a] In the opposite figure :

\overline{AB} and \overline{AC} are two tangent-segments to the circle M

, $m(\angle BMC) = 90^\circ$

Prove that : ABMC is a square.



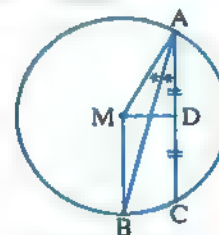
- [b] In the opposite figure :

\overline{AC} is a chord in the circle M

, \overline{AB} bisects $\angle CAM$

, D is the midpoint of \overline{AC}

Prove that : $\overline{DM} \perp \overline{MB}$



- 3 [a] In the opposite figure :

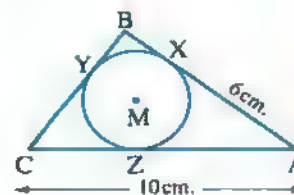
\overline{AB} , \overline{BC} and \overline{AC} are tangents to the circle M at X

, Y and Z respectively, $AC = 10$ cm.

, $AX = 6$ cm. and the perimeter of $\triangle ABC = 24$ cm.

- 1 Find : The length of \overline{AB}

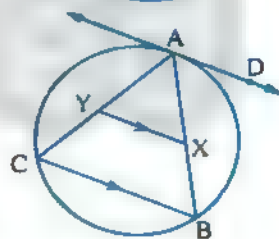
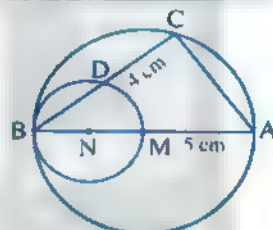
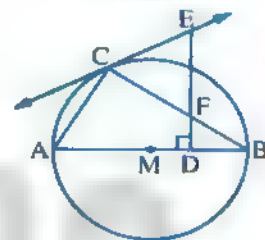
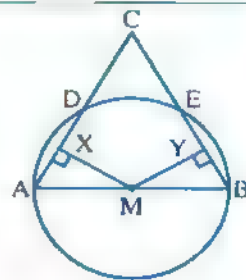
- 2 Determine the type of $\triangle ABC$ according to the measures of its angles.



[b] ABC is a triangle inscribed in a circle , $X \in \widehat{AB}$, $Y \in \widehat{AC}$ where $m(\widehat{AX}) = m(\widehat{AY})$, $\overline{CX} \cap \overline{AB} = \{D\}$ and $\overline{BY} \cap \overline{AC} = \{E\}$

Prove that : 1 The figure BCED is a cyclic quadrilateral.

2 $m(\angle DEB) = m(\angle XAB)$



7

El-Gharbia Governorate

Answer the following questions :

1 Choose the correct answer from those given :

1 A square whose diagonal length is 10 cm. , then its surface area equals cm²

(a) 40 (b) 50 (c) 80 (d) 100

2 ABC is a triangle in which $(AC)^2 > (AB)^2 + (BC)^2$, then $\angle BAC$ is

(a) acute. (b) obtuse. (c) right. (d) straight.

Geometry

- 3 M and N are two intersecting circles at two points and the two radii lengths are 3 cm. and 5 cm. , then $MN \in \dots\dots\dots$

(a) $]8, \infty[$ (b) $]2, \infty[$ (c) $]0, 2[$ (d) $]2, 8[$

- 4 ABCD is a cyclic quadrilateral in which $m(\angle A) = 3 m(\angle C)$, then $m(\angle A) = \dots\dots\dots^\circ$

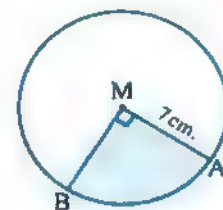
(a) 90 (b) 45 (c) 135 (d) 120

- 5 In the opposite figure :

\overline{MA} , \overline{MB} are two radii perpendicular in the circle M whose radius length is 7 cm.

, then the perimeter of the shaded part = $\dots\dots\dots$ cm. $(\pi = \frac{22}{7})$

(a) 14 (b) 11 (c) $38 \frac{1}{2}$ (d) 25



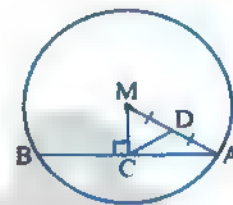
- 6 In the opposite figure :

\overline{AB} is a chord in the circle M

, $\overline{MC} \perp \overline{AB}$, D is the midpoint of \overline{MA} , $CD = 3$ cm.

, then the surface area of the circle M = $\dots\dots\dots \pi \text{ cm}^2$

(a) 3 (b) 6 (c) 9 (d) 36

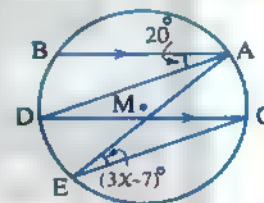


- 2 [a] In the opposite figure :

$\overline{AB} \parallel \overline{CD}$, $m(\angle BAD) = 20^\circ$

, $m(\angle AEC) = (3x - 7)^\circ$

What is the value of X ?



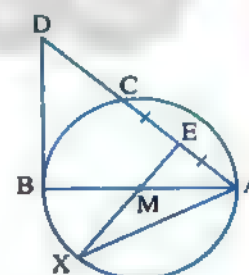
- [b] In the opposite figure :

\overline{AB} is a diameter in the circle M , \overline{BD} is a tangent-segment

to the circle M at B , E is the midpoint of \overline{AC} and \overline{EM} intersects the circle M at X

Prove that : 1 The figure MEDB is a cyclic quadrilateral.

2 $m(\angle BAX) = \frac{1}{2} m(\angle D)$



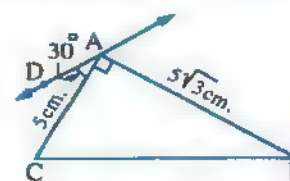
- 3 [a] In the opposite figure :

ABC is a right-angled triangle at A

, $AC = 5$ cm. , $AB = 5\sqrt{3}$ cm.

, $m(\angle DAC) = 30^\circ$

Prove that : \overline{AD} is a tangent to the circle passing through the vertices of $\triangle ABC$

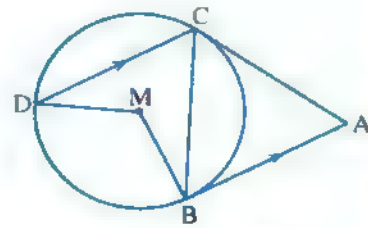


[b] In the opposite figure :

\overline{AB} , \overline{AC} are two tangent-segments to the circle M at B and C

, $\overline{AB} \parallel \overline{CD}$

Prove that : \overline{CB} bisects $\angle ACD$



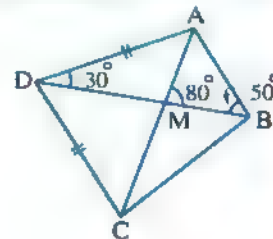
4 [a] In the opposite figure :

ABCD is a quadrilateral in which $\overline{AC} \cap \overline{BD} = \{M\}$, $DA = DC$

, $m(\angle ADM) = 30^\circ$, $m(\angle AMB) = 80^\circ$

, $m(\angle ABD) = 50^\circ$

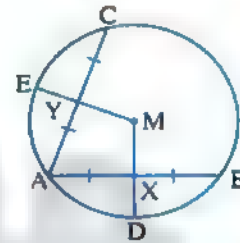
Prove that : The figure ABCD is a cyclic quadrilateral.



[b] In the opposite figure :

\overline{AB} , \overline{AC} are two chords equal in length in the circle M , X and Y are the midpoints of \overline{AB} and \overline{AC} respectively , \overline{MX} intersects the circle M at D , \overline{MY} intersects the circle M at E

Prove that : $XD = YE$



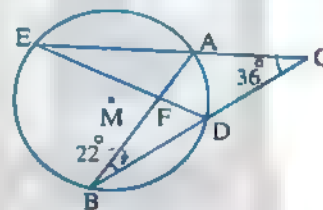
5 [a] In the opposite figure :

$\overline{EA} \cap \overline{BD} = \{C\}$

, $m(\angle C) = 36^\circ$

, $m(\angle ABD) = 22^\circ$

Find with the proof : $m(\widehat{BE})$



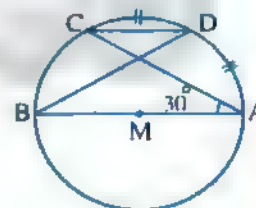
[b] In the opposite figure :

\overline{AB} is a diameter in the circle M

, $m(\angle CAB) = 30^\circ$, $m(\widehat{AD}) = m(\widehat{DC})$

1 Find with the proof : $m(\angle CDB)$

2 Prove that : $\overline{DC} \parallel \overline{AB}$



8

El-Dakahlia Governorate



Answer the following questions : (Calculator is permitted)

1 [a] Choose the correct answer from the given ones :

1 A circle with greatest chord with length = 12 cm. , then the circumference of the circle = cm.

(a) 12π

(b) 6π

(c) 24π

(d) 10π

Geometry

2 M and N are two circles whose radii lengths are 6 cm. , 8 cm. and $MN = 14$ cm. , then the two circles are

- (a) intersecting. (b) distant.
(c) one inside the other. (d) touching externally.

3 The inscribed angle drawn in a semicircle is angle.

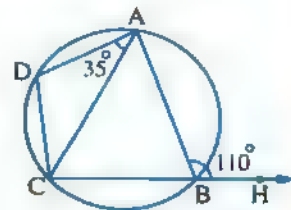
- (a) an acute (b) a straight (c) a right (d) an obtuse

[b] In the opposite figure :

$$m(\angle ABH) = 110^\circ$$

$$, m(\angle CAD) = 35^\circ$$

Prove that : $m(\widehat{CD}) = m(\widehat{AD})$



2 [a] Choose the correct answer from the given ones :

[1] A chord is of length 8 cm. in a circle of diameter length 10 cm. , then the chord is at from the center of the circle.

- (a) 2 cm. (b) 4 cm. (c) 3 cm. (d) 6 cm.

[2] The number of common tangents of two circles touching internally is

- (a) 1 (b) 3 (c) 2 (d) 0

[3] ABCD is a cyclic quadrilateral , $m(\angle A) = 2 m(\angle C)$, then $m(\angle A) =$

- (a) 30° (b) 60° (c) 90° (d) 120°

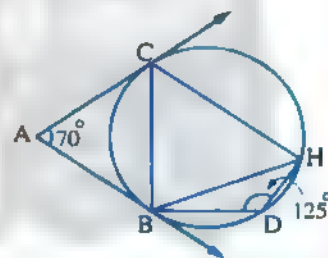
[b] In the opposite figure :

\overline{AB} and \overline{AC} are two tangents to the circle at B , C

$$, m(\angle A) = 70^\circ , m(\angle D) = 125^\circ$$

1 Find : $m(\angle ABC)$

2 Prove that : $CB = BH$



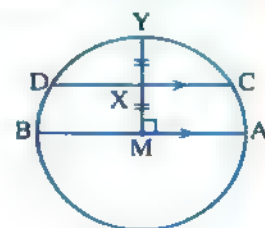
3 [a] In the opposite figure :

\overline{AB} is a diameter of the circle M , $\overline{CD} \parallel \overline{AB}$

, X is the midpoint of \overline{MY}

, $\overline{MY} \perp \overline{AB}$

Find : $m(\widehat{AC})$, $m(\widehat{CY})$



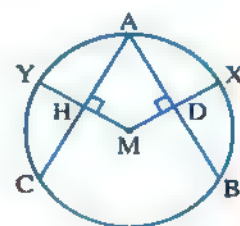
[b] In the opposite figure :

\overline{AB} , \overline{AC} are two equal chords in the circle M

, $\overline{MD} \perp \overline{AB}$ and cuts the circle at X

, $\overline{MH} \perp \overline{AC}$ and cuts the circle at Y

Prove that : $XD = HY$

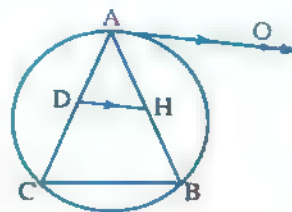


4 [a] In the opposite figure :

\overrightarrow{AO} is a tangent to the circle at A

, $\overrightarrow{AO} \parallel \overrightarrow{DH}$

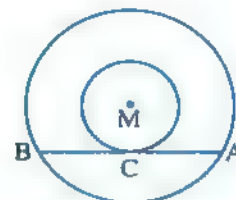
Prove that : DHBC is a cyclic quadrilateral.



[b] In the opposite figure :

\overline{AB} is a chord in the greater circle M and touches the smaller circle at C , if $AB = 14$ cm.

, find the area of the part included between the two circles.



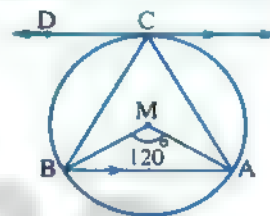
5 [a] In the opposite figure :

The circle M passes through the vertices of the triangle ABC

, $m(\angle AMB) = 120^\circ$

, \overrightarrow{CD} is a tangent to the circle M at C , $\overrightarrow{CD} \parallel \overrightarrow{AB}$

Prove that : $\triangle ABC$ is equilateral.

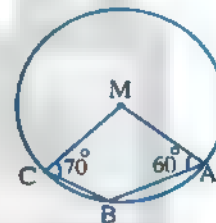


[b] In the opposite figure :

$m(\angle MAB) = 60^\circ$

, $m(\angle MCB) = 70^\circ$

Find : $m(\angle AMC)$



9

Ismailia Governorate



Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

1 The least number of acute angles at any triangle equals

(a) zero

(b) 1

(c) 2

(d) 3

2 The measure of the central angle drawn in $\frac{1}{3}$ circle equals

(a) 240

(b) 120

(c) 60

(d) 30

3 ABC is a triangle in which : $(AC)^2 = (AB)^2 + (BC)^2 + 5$, then $\angle B$ is

(a) acute.

(b) right.

(c) obtuse.

(d) straight.

4 Which of the following figures is a cyclic quadrilateral ?

(a) The square.

(b) The rhombus.

(c) The parallelogram.

(d) The trapezium.

Geometry

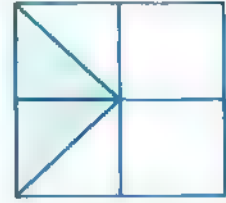
- 5 | If $AB = 8$ cm. , then the length of the radius of the smallest circle can be drawn passing through the two points A and B equals cm.

(a) 1 (b) 2 (c) 3 (d) 4

- 6 | In the opposite figure :

A square consists of congruent squares , then the area of the shaded part = the figure area.

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{3}{8}$ (d) $\frac{3}{4}$



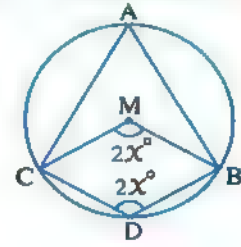
- 2 | [a] In the opposite figure :

\overline{AB} and \overline{AC} are two chords of the circle M

, $D \in \widehat{BC}$

, $m(\angle BMC) = m(\angle BDC) = (2x)^\circ$

Find with proof : $m(\angle A)$

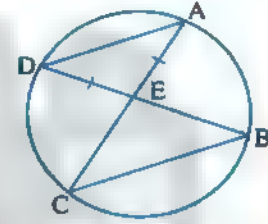


- [b] In the opposite figure :

$\overline{AC} \cap \overline{BD} = \{E\}$

, $EA = ED$

Prove that : $EB = EC$



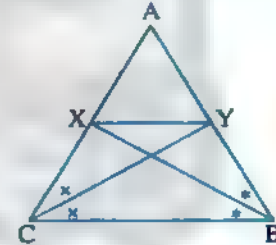
- 3 | [a] In the opposite figure :

ABC is a triangle in which $AB = AC$

, \overline{BX} bisects $\angle ABC$ and intersects \overline{AC} at X

, \overline{CY} bisects $\angle ACB$ and intersects \overline{AB} at Y

Prove that : BCXY is a cyclic quadrilateral.



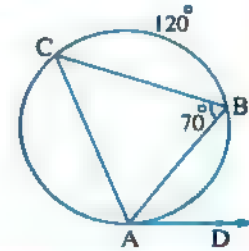
- [b] In the opposite figure :

\overline{AD} is a tangent to the circle at A

, $m(\angle B) = 70^\circ$

, $m(\widehat{BC}) = 120^\circ$

Find : $m(\angle DAB)$

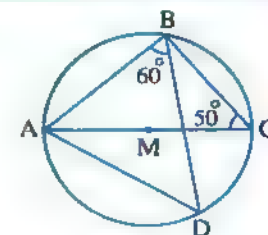


- 4 | [a] In the opposite figure :

\overline{AC} is a diameter of the circle M

, $m(\angle C) = 50^\circ$, $m(\angle ABD) = 60^\circ$

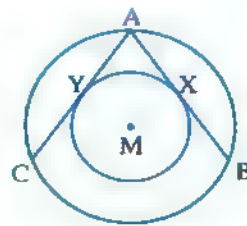
Find : $m(\angle CBD)$, $m(\angle BAD)$



[b] In the opposite figure :

Two concentric circles at M , \overline{AB} and \overline{AC} are two chords in the greater circle and two tangents to the smaller circle at X and Y respectively.

Prove that : $AB = AC$



5 [a] In the opposite figure :

Two circles are touching externally at C

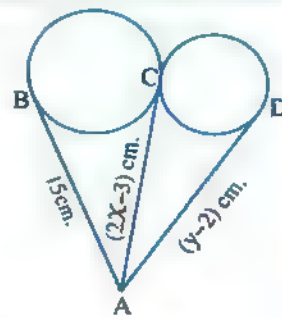
, \overline{AD} is a tangent-segment to the smaller circle at D

, \overline{AB} is a tangent-segment to the greater circle at B

If $AD = (y - 2)$ cm.

, $AC = (2x - 3)$ cm. , $AB = 15$ cm.

Find with proof : The value of each of x and y

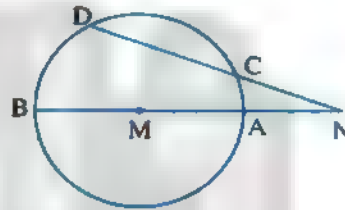


[b] In the opposite figure :

\overline{AB} is a diameter in the circle M

, $\overline{BA} \cap \overline{DC} = \{N\}$

Prove that : $NB > ND$



10

Suez Governorate



Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

1 The inscribed angle drawn in a semicircle is

(a) reflex.

(b) right.

(c) obtuse.

(d) acute.

2 In the opposite figure :

If M is a circle , $m(\angle AMB) = 80^\circ$

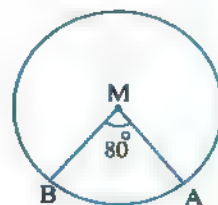
then $m(\widehat{AB}) = \dots^\circ$

(a) 40

(b) 80

(c) 160

(d) 90



3 If the two circles M , N are touching externally , the length of the radius of one of them is 3 cm. , $MN = 8$ cm. , then the length of the radius of the other circle is cm.

(a) 5

(b) 6

(c) 11

(d) 16

Geometry

[4] In the opposite figure :

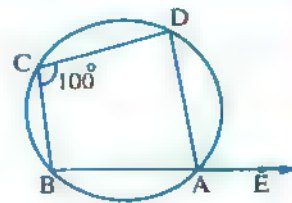
$E \in \overrightarrow{BA}$, $m(\angle C) = 100^\circ$
 , then $m(\angle DAE) = \dots\dots\dots^\circ$

(a) 80

(b) 60

(c) 100

(d) 200



[5] In the opposite figure :

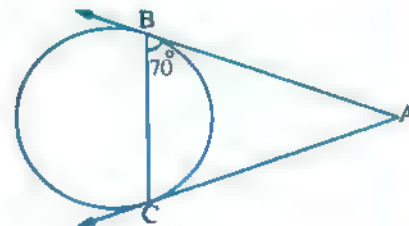
\overline{AB} and \overline{AC} are two tangents to
 the circle at B and C, $m(\angle ABC) = 70^\circ$
 , then $m(\angle A) = \dots\dots\dots^\circ$

(a) 80

(b) 70

(c) 60

(d) 40

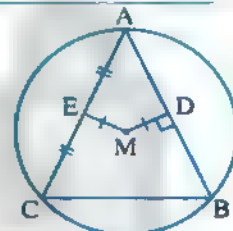


[6] The area of the circle =

(a) $2\pi r$ (b) πr^2 (c) $2\pi r^2$ (d) πr

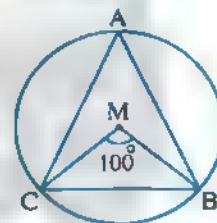
[2] [a] In the opposite figure :

If M is a circle, $\overline{MD} \perp \overline{AB}$
 , E is the midpoint of \overline{AC}
 , $MD = ME$
 , prove that : $AB = AC$



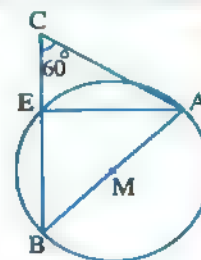
[b] In the opposite figure :

If M is a circle, $m(\angle BMC) = 100^\circ$
 , find : [1] $m(\angle A)$
 [2] $m(\angle MBC)$



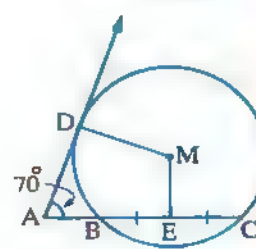
[3] [a] In the opposite figure :

\overline{AB} is a diameter of the circle M
 , $C \in \overline{BE}$, $m(\angle ACE) = 60^\circ$
 Find : [1] $m(\angle AEB)$
 [2] $m(\angle CAE)$



[b] In the opposite figure :

\overline{AD} is a tangent to the circle M
 , \overline{AC} intersects the circle M at B, C
 , E is the midpoint of \overline{BC} , $m(\angle A) = 70^\circ$
 Find : $m(\angle DME)$



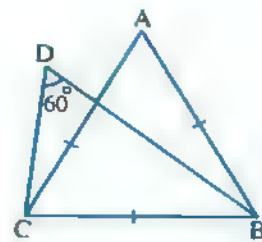
4 [a] State two cases of the cyclic quadrilateral.

[b] In the opposite figure :

ABC is an equilateral triangle

, $m(\angle D) = 60^\circ$

Prove that : ABCD is a cyclic quadrilateral.



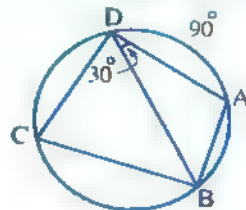
5 [a] In the opposite figure :

In the circle , $m(\angle ADB) = 30^\circ$

, $m(\widehat{AD}) = 90^\circ$

Find : 1 $m(\widehat{AB})$

2 $m(\angle DCB)$



[b] In the opposite figure :

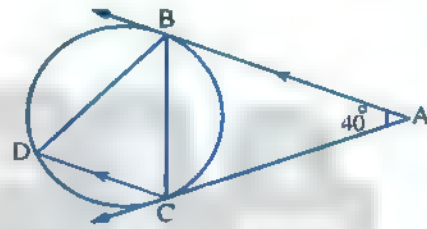
\overline{AB} and \overline{AC} are two tangents

to the circle at B and C

, $\overline{AB} \parallel \overline{CD}$, $m(\angle A) = 40^\circ$

1 Find : $m(\angle ABC)$

2 Prove that : $BC = BD$



11 Port Said Governorate



Answer the following questions :

1 Choose the correct answer from those given :

1 M and N are two intersecting circles. The two radii lengths are 3 cm. and 5 cm. respectively , then $MN \in \dots\dots\dots$

(a) $]8, \infty[$

(b) $]2, \infty[$

(c) $]0, 2[$

(d) $]2, 8[$

2 If the straight line L is a tangent to the circle M of diameter length 10 cm. , then the distance between L and the center of the circle equals cm.

(a) 3

(b) 4

(c) 5

(d) 10

3 The longest chord in the circle is called a

(a) chord.

(b) diameter.

(c) tangent.

(d) radius.

4 In the opposite figure :

If $m(\angle A) = 120^\circ$

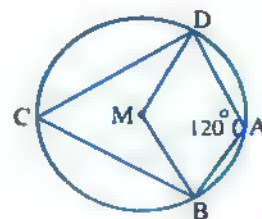
, then $m(\angle DMB) = \dots\dots\dots$

(a) 180°

(b) 120°

(c) 90°

(d) 60°



Geometry

- 5 The ratio between the measure of the central angle and the measure of the inscribed angle subtended by the same arc is

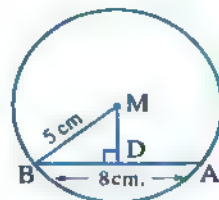
(a) 4 : 2 (b) 2 : 4 (c) 3 : 2 (d) 2 : 3

- 6 In the opposite figure :

AB = 8 cm. , MB = 5 cm.

, then MD =

(a) 5 cm. (b) 3 cm.
(c) 4 cm. (d) 2 cm.



- 2 [a] In the opposite figure :

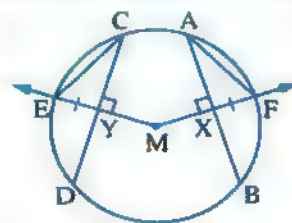
AB and CD are two chords in the circle M

, $\overline{MX} \perp \overline{AB}$ and intersects the circle at F

, $\overline{MY} \perp \overline{CD}$ and intersects the circle at E , $FX = EY$

Prove that : 1 AB = CD

2 AF = CE



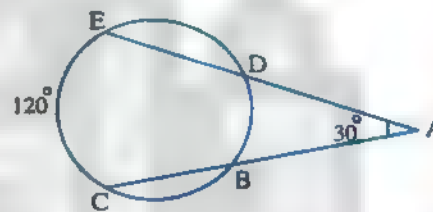
- [b] In the opposite figure :

$\overline{ED} \cap \overline{CB} = \{A\}$

, $m(\widehat{CE}) = 120^\circ$

, $m(\angle A) = 30^\circ$

Find : $m(\widehat{BD})$

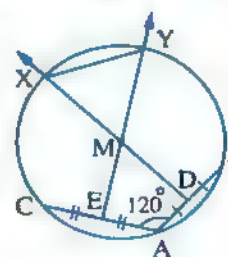
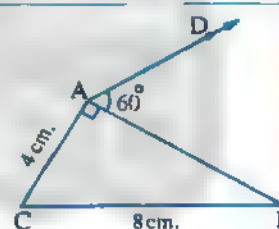


- 3 [a] Using the given data , prove that :

\overline{AD} is a tangent to the circle passing through the vertices of the triangle ABC

- [b] Using the given data , prove that :

The triangle XYM is an equilateral triangle.



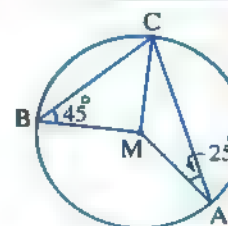
- 4 [a] In the opposite figure :

A circle with center M

, $m(\angle MAC) = 25^\circ$

, $m(\angle MBC) = 45^\circ$

Find : $m(\angle AMB)$

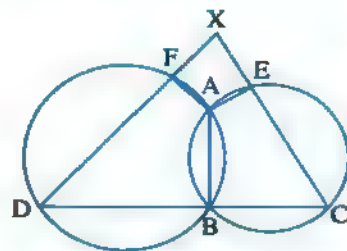


[b] In the opposite figure :

Two intersecting circles at A and B , \overline{CD} passes through the point B and intersects the two circles at C and D

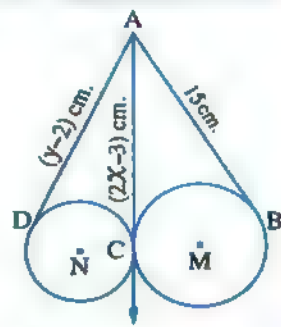
$$\overrightarrow{CE} \cap \overrightarrow{DF} = \{X\}$$

Prove that : The figure AFXE is a cyclic quadrilateral.



5 [a] Using the given data , find :

The values of the symbols X and y

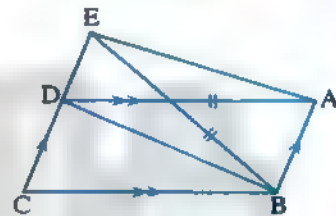


[b] In the opposite figure :

ABCD is a parallelogram

, $E \in \overline{CD}$ where $BE = AD$

Prove that : The figure ABDE is a cyclic quadrilateral.



12

Damietta Governorate



Answer the following questions : (Calculator is allowed)

1 Choose the correct answer :

- 1 The length of the projection of a line segment on a given straight line the length of the line segment.
(a) > (b) \leq (c) \geq (d) <
- 2 The number of symmetry axes of any circle is
(a) zero (b) 1 (c) 2 (d) an infinite number
- 3 If a square is of side length 6 cm. , then the square of its diagonal length is cm^2
(a) 36 (b) 12 (c) 72 (d) $6\sqrt{2}$
- 4 If the straight line L is a tangent to the circle M of diameter length 10 cm. , then the distance between L and the center of the circle equals cm.
(a) 3 (b) 5 (c) 6 (d) 10
- 5 If M , N are two touching circles internally , their radii lengths are 7 cm. , 10 cm. , then MN = cm.
(a) 3 (b) 17 (c) 7 (d) 10

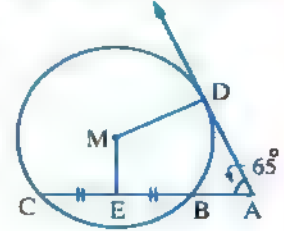
Geometry

- 6 If $\Delta XYZ \sim \Delta ABC$, $m(\angle Y) = 60^\circ$ and $m(\angle C) = 40^\circ$, then $m(\angle X) = \dots\dots\dots^\circ$
 (a) 40 (b) 80 (c) 100 (d) 120

- 2 [a] In the opposite figure :

\overline{AD} is a tangent to the circle M, \overline{AC} intersects the circle at B, C
 , E is the midpoint of \overline{BC}
 , $m(\angle A) = 65^\circ$

Find : $m(\angle DME)$



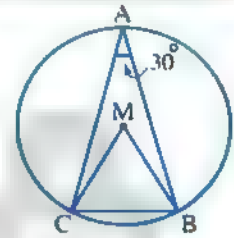
- [b] If the length of $\overline{AB} = 6$ cm. , draw a circle of radius length 4 cm. that passes through A , B
 How many circles can be drawn ? (Don't remove the arcs).

- 3 [a] In the opposite figure :

A circle M , $m(\angle A) = 30^\circ$

1 Find : $m(\angle BMC)$

2 Prove that : MBC is an equilateral triangle.



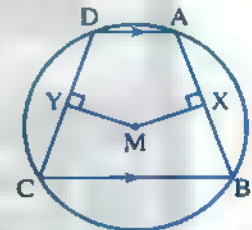
- [b] In the opposite figure :

A circle M , $\overline{AD} \parallel \overline{BC}$

, $\overline{MX} \perp \overline{AB}$

, $\overline{MY} \perp \overline{DC}$

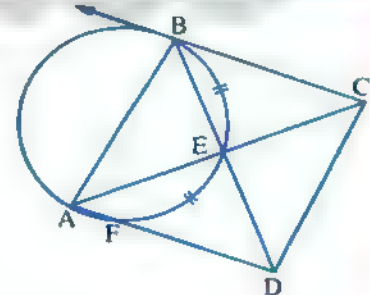
Prove that : $MX = MY$



- 4 [a] In the opposite figure :

\overline{CB} is a tangent , $m(\widehat{BE}) = m(\widehat{EF})$

Prove that : ABCD is a cyclic quadrilateral.



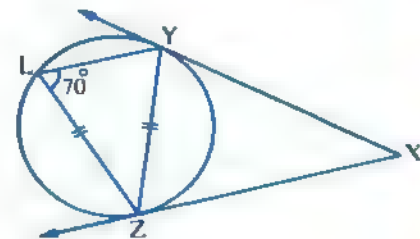
- [b] In the opposite figure :

\overline{XY} , \overline{XZ} are two tangents to the circle at Y , Z

, $YZ = LZ$, $m(\angle L) = 70^\circ$

1 Find with proof : $m(\angle X)$

2 Prove that : $\overline{XZ} \parallel \overline{YL}$



- 5 [a] ABCD is a parallelogram in which $AC = BC$

Prove that : \overline{CD} is a tangent to the circle circumscribed about the triangle ABC

- [b] In the opposite figure :

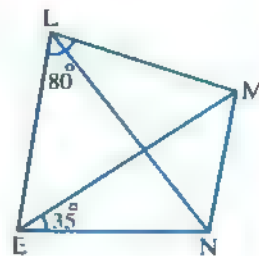
LMNE is a cyclic quadrilateral , $m(\angle MEN) = 35^\circ$

, $m(\angle MLE) = 80^\circ$

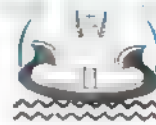
Find with proof :

1 $m(\angle MLN)$

2 $m(\angle EMN)$



13 Kafr El-Sheikh Governorate



Answer the following questions : (Calculator is allowed)

- 1 Choose the correct answer from those given :

- 1 The triangle contains two angles at least.

(a) acute

(b) obtuse

(c) right

(d) reflex

- 2 ABCD is a rhombus in which $m(\angle ACB) = 32^\circ$, then $m(\angle D) =$

(a) 32°

(b) 64°

(c) 116°

(d) 26°

- 3 A tangent to a circle of diameter length 6 cm. is at a distance of cm. from its center.

(a) 6

(b) 12

(c) 3

(d) 2

- 4 If M , N are two touching circles internally their radii lengths are 8 cm. , 3 cm. , then $MN =$ cm.

(a) 3

(b) 5

(c) 7

(d) 11

- 5 The triangle whose side lengths are 5 cm. , 7 cm. and 8 cm. is triangle.

(a) obtuse-angled.

(b) acute-angled.

(c) right-angled.

(d) equilateral.

- 6 The number of common tangents to two touching circles externally is

(a) 0

(b) 1

(c) 2

(d) 3

- 2 [a] In the opposite figure :

\overline{AB} and \overline{BC} are two chords in the circle M

, which has radius length of 10 cm.

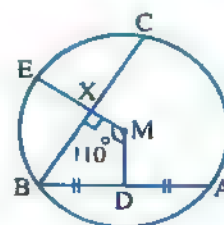
, $\overline{MX} \perp \overline{BC}$ intersecting \overline{BC} at X and intersecting the circle M at E

, D is the midpoint of \overline{AB} , $BC = 16$ cm.

, $m(\angle DMX) = 110^\circ$

Find : 1 The length of \overline{XE}

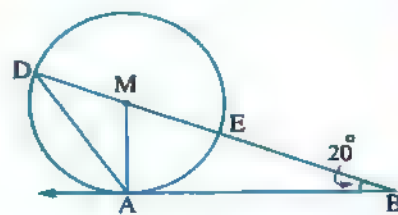
2 $m(\angle ABC)$



Geometry

[b] In the opposite figure :

B is a point outside the circle M

, \overrightarrow{BA} is a tangent to the circle M at A, \overrightarrow{BM} intersects the circle at E and D , $m(\angle B) = 20^\circ$ Find with proof : $m(\angle ADB)$ 

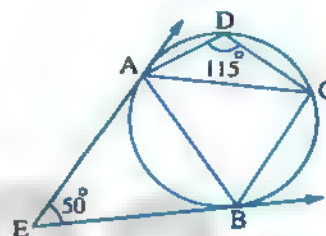
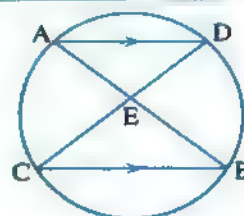
3 [a] In the opposite figure :

 $\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$, $\overrightarrow{AD} \parallel \overrightarrow{CB}$ Prove that : $EA = ED$

[b] In the opposite figure :

 \overrightarrow{EA} and \overrightarrow{EB} are two tangents to the circle at A , B, $m(\angle AEB) = 50^\circ$, $m(\angle ADC) = 115^\circ$

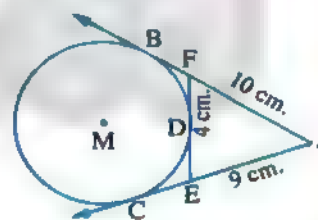
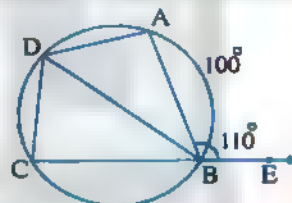
Prove that :

 \overrightarrow{AC} is a tangent to the circle passing through the points A , B and E

4 [a] In the opposite figure :

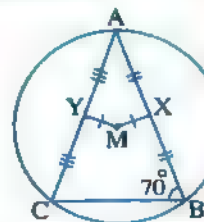
 $E \in \overrightarrow{CB}$, $m(\widehat{AB}) = 100^\circ$, $m(\angle ABE) = 110^\circ$ Find with proof : $m(\angle BDC)$

[b] In the opposite figure :

 \overrightarrow{AB} and \overrightarrow{AC} are two tangents to the circle M at B , C, \overrightarrow{FE} is a tangent-segment at D , $DF = 4$ cm., $AF = 10$ cm. , $AE = 9$ cm.Find with proof : The length of \overrightarrow{EC} 

5 [a] In the opposite figure :

ABC is an inscribed triangle inside the circle M

, $MX = MY$, X and Y are the midpoints of \overrightarrow{AB} , \overrightarrow{AC} respectively , $m(\angle B) = 70^\circ$ Find with proof : $m(\angle A)$ [b] ABC is an inscribed triangle in a circle where $AB > AC$ and $D \in \overrightarrow{AB}$ where $AC = AD$, \overrightarrow{AE} bisects $\angle A$ and intersects \overrightarrow{BC} at E and intersects the circle at F

Prove that : BDEF is a cyclic quadrilateral.

14 El-Beheira Governorate



Answer the following questions : (Calculator is permitted)

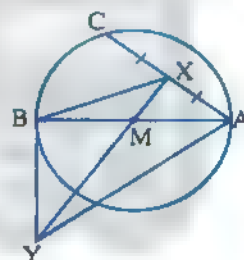
1 Choose the correct answer from the given ones :

- 1 M and N are two intersecting circles , their radii lengths are 3 cm. and 5 cm. , then $MN \in \dots\dots\dots$
 (a) $]8, \infty[$ (b) $]2, \infty[$ (c) $]0, 2[$ (d) $]2, 8[$
- 2 ABCD is a cyclic quadrilateral , $m(\angle A) = 70^\circ$, then $m(\angle C)$ equals
 (a) 25° (b) 20° (c) 110° (d) 100°
- 3 The measure of the inscribed angle drawn in a semicircle equals
 (a) 130° (b) 90° (c) 50° (d) 180°
- 4 The slope of the straight line $3x + 2y = 1$ is
 (a) $\frac{2}{3}$ (b) $-\frac{3}{2}$ (c) $-\frac{2}{3}$ (d) $\frac{3}{2}$
- 5 The measurement of any angle of the regular hexagon is
 (a) 90° (b) 108° (c) 120° (d) 135°
- 6 In $\triangle ABC$, if $(AB)^2 = (AC)^2 + (BC)^2$, then $\angle B$ is
 (a) acute. (b) obtuse. (c) right. (d) reflex.

2 [a] In the opposite figure :

\overline{AB} is a diameter in the circle M
 , X is the midpoint of \overline{AC} and \overline{XM} intersects
 the tangent to the circle at B in Y

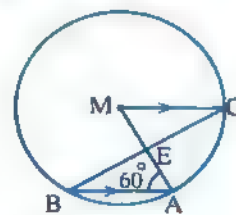
Prove that : The figure AXBY is a cyclic quadrilateral.



[b] In the opposite figure :

\overline{AB} is a chord in the circle M
 , $\overline{CM} \parallel \overline{AB}$, $\overline{BC} \cap \overline{AM} = \{E\}$
 , $m(\angle A) = 60^\circ$

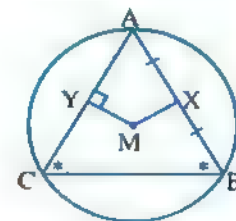
Find : $m(\angle B)$



3 [a] In the opposite figure :

The triangle ABC is inscribed in the circle M
 , in which : $m(\angle B) = m(\angle C)$
 , X is the midpoint of \overline{AB} , $\overline{MY} \perp \overline{AC}$

Prove that : $MX = MY$



Geometry

[b] In the opposite figure :

ABCDE is a regular pentagon inscribed in a circle M

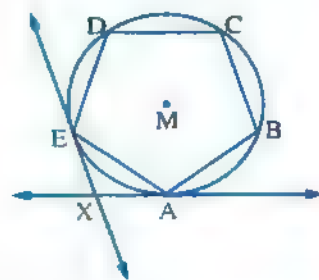
, \overrightarrow{AX} is a tangent to the circle at A

, \overrightarrow{EX} is a tangent to the circle at E

where $\overrightarrow{AX} \cap \overrightarrow{EX} = \{X\}$

Find : 1 $m(\widehat{AE})$

2 $m(\angle AXE)$



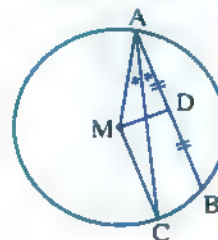
4 [a] In the opposite figure :

\overline{AB} is a chord in the circle M

, \overline{AC} bisects $\angle BAM$ and intersects the circle M at C

If D is the midpoint of \overline{AB}

, prove that : $\overline{DM} \perp \overline{CM}$



[b] \overline{AB} is a diameter in the circle M , \overline{AC} and \overline{BD} are two tangents to the circle M , \overline{CM} intersects the circle M at X and Y and intersects \overline{BD} at E Prove that : $CX = YE$

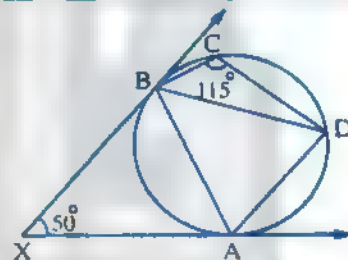
5 [a] In the opposite figure :

\overline{XA} and \overline{XB} are two tangents to the circle at A and B

, $m(\angle AXB) = 50^\circ$, $m(\angle DCB) = 115^\circ$

Prove that : 1 \overline{AB} bisects $\angle DAX$

2 $BD = BA$

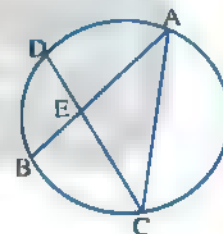


[b] In the opposite figure :

\overline{AB} and \overline{CD} are two equal chords in length in the circle

, $\overline{AB} \cap \overline{CD} = \{E\}$

Prove that : The triangle ACE is an isosceles triangle.



15 El-Fayoum Governorate



Answer the following questions : (Using calculators is allowed)

1 Choose the correct answer :

- 1 If M is a circle of diameter length 8 cm. , the straight line L is far from the centre M of the circle by 4 cm. , then the straight line L is
- (a) a secant to the circle in two points. (b) outside the circle.
- (c) a tangent to the circle. (d) an axis of symmetry of the circle.

- 2 If m_1 , m_2 are the slopes of two perpendicular straight lines, then
- (a) $m_1 = m_2$ (b) $m_1 \times m_2 = -1$ (c) $m_1 \times m_2 = 1$ (d) $m_1 + m_2 = -1$
- 3 The centre of the circle that passes through the vertices of the triangle is the intersection point of
- (a) the bisectors of its interior angles. (b) the bisectors of its exterior angles.
(c) its altitudes. (d) the axes of its sides.
- 4 ABC is a right-angled triangle at B, $m(\angle C) = 30^\circ$, $AC = 12$ cm.
then $AB = \dots\dots\dots$ cm.
- (a) 24 (b) $12\sqrt{3}$ (c) $6\sqrt{3}$ (d) 6
- 5 Which of the following figures is a cyclic quadrilateral ?
- (a) The rectangle. (b) The trapezium. (c) The rhombus. (d) The parallelogram.
- 6 A trapezium in which the lengths of the two parallel bases are 4 cm. and 12 cm. and its height is 9 cm. , then its area = cm^2
- (a) 25 (b) 36 (c) 72 (d) 144

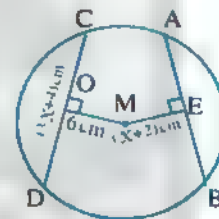
2 [a] In the opposite figure :

$AB = CD$, $MO = 6$ cm.

, $ME = (X + 2)$ cm.

, $CD = (3X + 4)$ cm.

Find : The value of X , CD



[b] ABC is a triangle drawn inside a circle M, $m(\angle AMB) = 90^\circ$, $m(\angle BMC) = 130^\circ$

Find : The measures of the angles of $\triangle ABC$

3 [a] A is a point outside the circle M, \overline{AB} is a tangent to the circle at B, \overline{AM} intersects the circle M at C and D respectively, $m(\angle A) = 40^\circ$

Find with proof : $m(\angle BDC)$

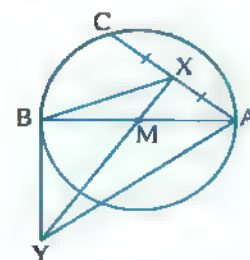
[b] In the opposite figure :

\overline{AB} is a diameter in the circle M

, X is the midpoint of \overline{AC}

and \overline{XM} intersects the tangent to the circle at B at Y

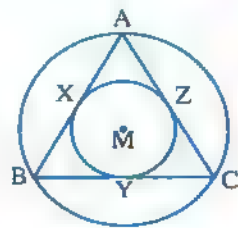
Prove that : The figure AXBY is a cyclic quadrilateral.



Geometry

4 [a] In the opposite figure :

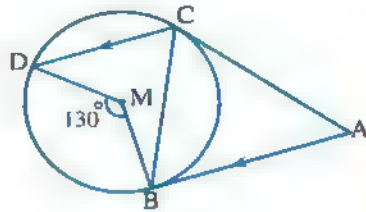
- Two concentric circles with centre M
 , the radii lengths of them are 4 cm. and 2 cm.
 , $\triangle ABC$ is an inscribed triangle inside the greater circle
 , and its sides touch the smaller circle at X , Y , Z



Prove that : $\triangle ABC$ is an equilateral triangle , and calculate its area.

[b] In the opposite figure :

- \overline{AB} , \overline{AC} are two tangent-segments to the circle M
 , $\overline{AB} \parallel \overline{CD}$
 , $m(\angle BMD) = 130^\circ$



Prove that : \overline{CB} bisects $\angle ACD$

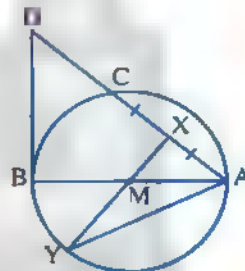
5 [a] $\triangle ABC$ is a triangle inscribed in a circle , \overline{AD} is a tangent to the circle at A , $X \in \overline{AB}$ and $Y \in \overline{AC}$, where $\overline{XY} \parallel \overline{BC}$

Prove that : \overline{AD} is a tangent to the circle passing through the points A , X and Y

[b] In the opposite figure :

- \overline{AB} is a diameter in the circle M ,
 X is the midpoint of \overline{AC} , \overline{BD} is a tangent to the circle at B , \overline{XM} intersects the circle at Y
Prove that : 1 XMBD is a cyclic quadrilateral.

$$2 m(\angle BAY) = \frac{1}{2} m(\angle D)$$



16 Beni Suef Governorate



Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

- 1 The symmetry axis of the common chord \overline{AB} of the two intersecting circles M , N is
 (a) \overline{MA} (b) \overline{MB} (c) \overline{MN} (d) \overline{NA}
- 2 ABC is a triangle in which : $(AC)^2 > (AB)^2 + (BC)^2$, then $\angle B$ is
 (a) acute. (b) obtuse. (c) right. (d) straight.
- 3 In the cyclic quadrilateral , each two opposite angles are
 (a) equal in measure. (b) complementary.
 (c) supplementary. (d) alternate.

- 4 The area of a triangle is 35 cm^2 and its height is 7 cm. , then the length of its base equals cm.

(a) 5 (b) 7 (c) 10 (d) 20

- 5 The measure of the inscribed angle which is drawn in a semicircle equals

(a) 45° (b) 90° (c) 120° (d) 180°

- 6 The area of a square is 100 cm^2 , then its perimeter = cm.

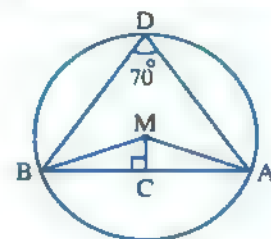
(a) 10 (b) 30 (c) 40 (d) 50

- 2 [a] In the opposite figure :

\overline{AB} is a chord in the circle M

, $\overline{MC} \perp \overline{AB}$, $m(\angle ADB) = 70^\circ$

Find : $m(\angle AMC)$

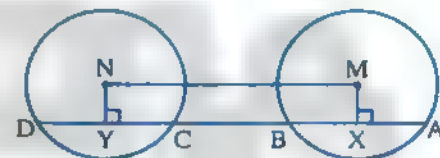


- [b] In the opposite figure :

M and N are two congruent circles

, $AB = CD$, $\overline{MX} \perp \overline{AB}$ and $\overline{NY} \perp \overline{CD}$

Prove that : The figure MXYN is a rectangle.



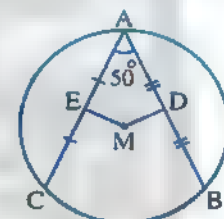
- 3 [a] In the opposite figure :

\overline{AB} and \overline{AC} are two chords

in the circle M , D is the midpoint of \overline{AB}

, E is the midpoint of \overline{AC} and $m(\angle BAC) = 50^\circ$

Find : $m(\angle DME)$



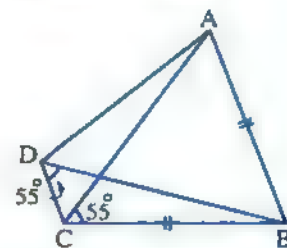
- [b] In the opposite figure :

$AB = BC$

, $m(\angle ACB) = 55^\circ$

and $m(\angle BDC) = 55^\circ$

Prove that : The figure ABCD is a cyclic quadrilateral.

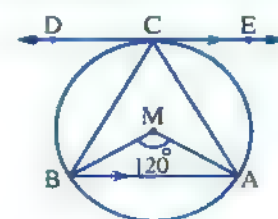


- 4 [a] In the opposite figure :

\overline{ED} is a tangent to the circle M at C

, $\overline{ED} \parallel \overline{AB}$ and $m(\angle AMB) = 120^\circ$

Prove that : The triangle CAB is an equilateral triangle.

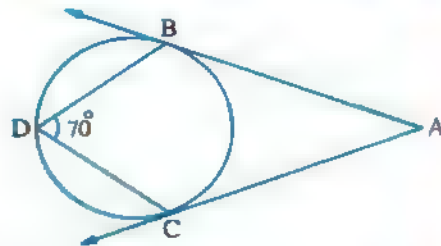


Geometry

[b] In the opposite figure :

\overline{AB} and \overline{AC} are two tangents to the circle at B and C

$$, m(\angle BDC) = 70^\circ$$

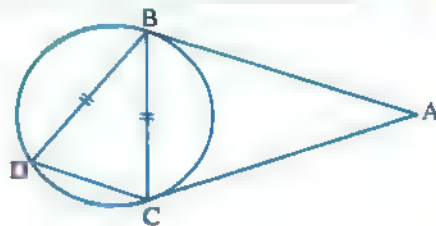
Find : $m(\angle A)$ 

5 [a] In the opposite figure :

\overline{AB} and \overline{AC} are two tangent-segments to the circle at B and C , $BC = BD$

Prove that :

\overline{BD} is a tangent to the circle passing through the vertices of $\triangle ABC$



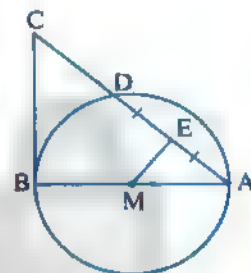
[b] In the opposite figure :

\overline{AB} is a diameter in the circle M

, \overline{BC} is a tangent to the circle

at B and E is the midpoint of \overline{AD}

Prove that : The figure EMBC is a cyclic quadrilateral.



17

El-Menia Governorate



Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

1] It is possible to draw a circle passing through the vertices of a

- (a) rhombus. (b) rectangle. (c) right trapezium. (d) parallelogram.

2] The inscribed angle drawn in a semicircle is

- (a) acute. (b) obtuse. (c) straight. (d) right.

3] The number of rectangles in the opposite figure is

- (a) 3 (b) 6 (c) 7 (d) 10

4] If the perimeter of a square is 20 cm. , then its surface area is cm^2

- (a) 20 (b) 25 (c) 50 (d) 100

5] The measure of the exterior angle of an equilateral triangle equals $^\circ$

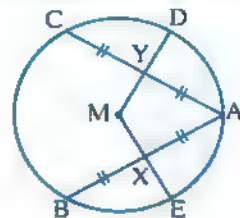
- (a) 60 (b) 108 (c) 120 (d) 135

- 6 If ABCD is a cyclic quadrilateral , $2m(\angle A) = 120^\circ$, then $m(\angle C) = \dots\dots\dots^\circ$
 (a) 120 (b) 45 (c) 60 (d) 90

2 [a] In the opposite figure :

\overline{AB} and \overline{AC} are two chords equal in length in the circle M , X is the midpoint of \overline{AB} and Y is the midpoint of \overline{AC}

Prove that : $XE = YD$



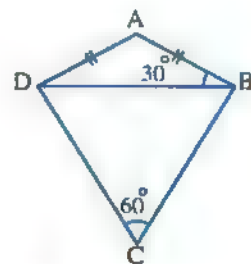
[b] In the opposite figure :

ABCD is a quadrilateral , $AB = AD$

, $m(\angle ABD) = 30^\circ$

, $m(\angle C) = 60^\circ$

Prove that : ABCD is a cyclic quadrilateral.



3 [a] In the opposite figure :

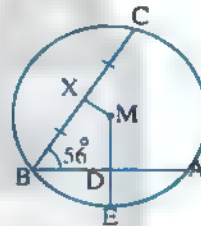
\overline{AB} and \overline{BC} are two chords in the circle M which has radius length of 5 cm. , $\overline{MD} \perp \overline{AB}$

, X is the midpoint of \overline{BC}

, $AB = 8$ cm. , $m(\angle B) = 56^\circ$

Find : 1 $m(\angle DMX)$

2 The length of \overline{DE}



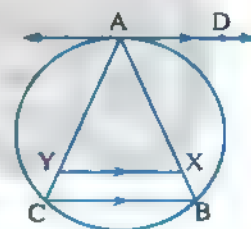
[b] In the opposite figure :

\overline{AD} is a tangent to the circle at A

, $X \in \overline{AB}$, $Y \in \overline{AC}$ where $\overline{XY} \parallel \overline{BC}$

Prove that :

\overline{AD} is a tangent to the circle passing through the points A , X and Y



4 [a] In the opposite figure :

$AB = AC$

, $E \in \widehat{BC}$

Prove that : $m(\angle AEB) = m(\angle AEC)$

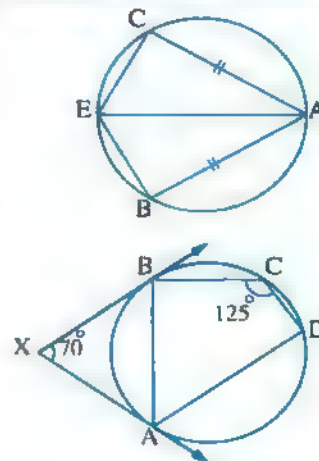
[b] In the opposite figure :

\overline{XA} and \overline{XB} are two tangents to the circle at A and B

, $m(\angle AXB) = 70^\circ$

, $m(\angle DCB) = 125^\circ$

Prove that : $m(\angle DAB) = m(\angle XAB)$



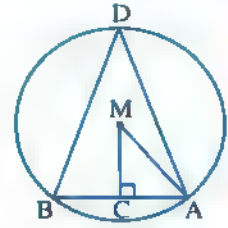
Geometry

5 [a] In the opposite figure :

\overline{AB} is a chord in the circle M

, $\overline{MC} \perp \overline{AB}$

Prove that : $m(\angle AMC) = m(\angle ADB)$



[b] ABC is an inscribed triangle in a circle M where $AB > AC$ and $D \in \overline{AB}$ where $AC = AD$, \overline{AE} bisects $\angle A$ and intersects \overline{BC} at E and intersects the circle at F

Prove that : BDEF is a cyclic quadrilateral.

18 Assiut Governorate



Answer the following questions : (Calculator is permitted)

1 Choose the correct answer :

1 XYZ is a triangle in which : D is the midpoint of \overline{XY} , E is the midpoint of \overline{XZ} , then $DE = \dots\dots\dots YZ$

- (a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 2

2 The diameter is a passing through the center of the circle.

- (a) straight line (b) ray (c) tangent (d) chord

3 If the circumference of a circle is 18π cm., then its radius length = cm.

- (a) 7 (b) 9 (c) 3 (d) 6

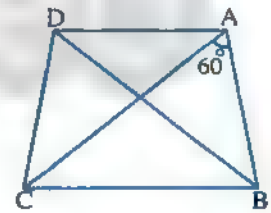
4 In the opposite figure :

ABCD is a cyclic quadrilateral

, $m(\angle BAC) = 60^\circ$

, then $m(\angle BDC) = \dots\dots\dots$

- (a) 300° (b) 120°
(c) 60° (d) 30°



5 The area of the triangle which the length of its base is 9 cm., its height is 12 cm. equals cm^2

- (a) 48 (b) 24 (c) 36 (d) 54

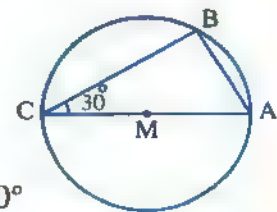
6 In the opposite figure :

\overline{AC} is a diameter in the circle M

, $m(\angle C) = 30^\circ$

, then $m(\angle A) = \dots\dots\dots$

- (a) 120° (b) 60° (c) 90° (d) 40°



2 [a] In the opposite figure :

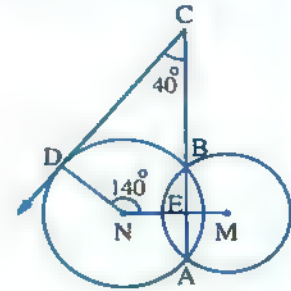
M and N are two intersecting circles at A and B

, $C \in \overline{AB}$, $\overline{AC} \cap \overline{MN} = \{E\}$

, $D \in \text{the circle N}$, $m(\angle DNM) = 140^\circ$

and $m(\angle C) = 40^\circ$

Prove that : \overline{CD} is a tangent to the circle N at D

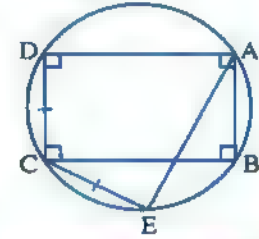
**[b] In the opposite figure :**

ABCD is a rectangle inscribed in a circle

, the chord \overline{CE} is drawn

where $CE = CD$

Prove that : $AE = BC$

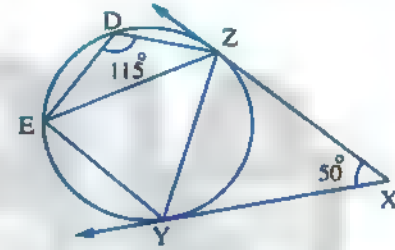
**3 [a] State two cases of the cyclic quadrilateral.****[b] In the opposite figure :**

\overline{XY} , \overline{XZ} are two tangents to the circle at Y, Z

, $m(\angle D) = 115^\circ$

and $m(\angle X) = 50^\circ$

Prove that : $ZE = ZY$

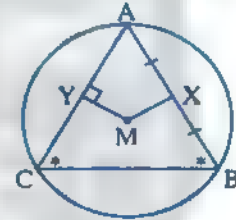
**4 [a] In the opposite figure :**

ABC is a triangle inscribed in the circle M

, in which $m(\angle B) = m(\angle C)$

, X is the midpoint of \overline{AB} , $\overline{MY} \perp \overline{AC}$

Prove that : $MX = MY$

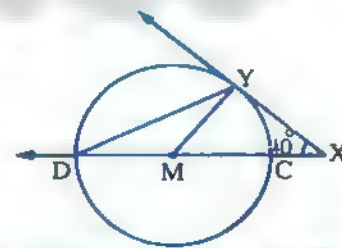
**[b] In the opposite figure :**

X is a point outside the circle M, \overline{XY} is a tangent

to the circle at Y, \overline{XM} intersects the circle M at C

and D respectively, $m(\angle X) = 40^\circ$

Find : $m(\angle YDC)$

**5 [a] In the opposite figure :**

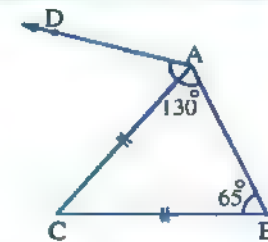
ABC is a triangle, $CB = AC$

, $m(\angle DAB) = 130^\circ$

, $m(\angle B) = 65^\circ$

Prove that :

\overline{AD} is a tangent to the circle passing through the vertices of the triangle ABC



Geometry

[b] In the opposite figure :

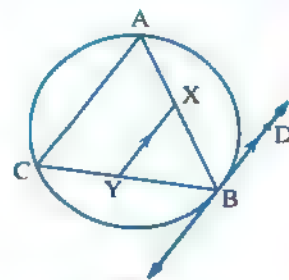
ABC is a triangle inscribed in a circle

, \overline{BD} is a tangent to the circle at B

, $X \in \overline{AB}$, $Y \in \overline{BC}$

where $\overline{XY} \parallel \overline{BD}$

Prove that : AXYC is a cyclic quadrilateral.



19

Souhag Governorate



Answer the following questions : (Calculator is permitted)

1 Choose the correct answer :

- 1 If the straight line L is a tangent to the circle M of diameter length 8 cm. , then the distance between L and the center of the circle equals cm.

(a) 3 (b) 4 (c) 6 (d) 8

- 2 The area of the rhombus = of the product of the lengths of its diagonals.

(a) 2 (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 3

- 3 The number of symmetry axes of any circle is

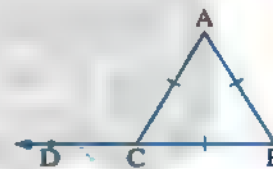
(a) zero (b) 1 (c) 2 (d) an infinite number.

4 In the opposite figure :

The triangle ABC is an equilateral triangle

, then $m(\angle ACD) = \dots\dots\dots^\circ$

(a) 45 (b) 60
(c) 120 (d) 135



- 5 If the lengths of two sides of an isosceles triangle are 2 cm. and $(X + 3)$ cm. , and the length of the third side is 5 cm. , then $X = \dots\dots\dots$ cm.

(a) 1 (b) 2 (c) 3 (d) 4

- 6 If M , N are two touching circles internally , their radii lengths are 5 cm. , 9 cm. , then $MN = \dots\dots\dots$ cm.

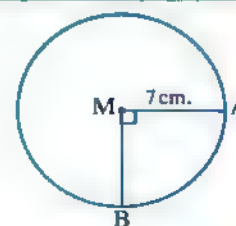
(a) 14 (b) 4 (c) 5 (d) 9

2 [a] In the opposite figure :

M is a circle with radius length 7 cm.

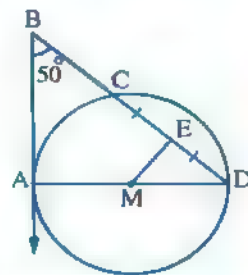
, $m(\angle AMB) = 90^\circ$

Find : The length of \widehat{AB} ($\pi = \frac{22}{7}$)



[b] In the opposite figure :

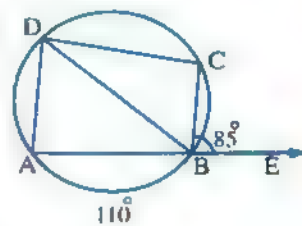
- \overline{AD} is a diameter in the circle M
 \overline{AB} is a tangent , $m(\angle B) = 50^\circ$
 E is the midpoint of \overline{DC}
 Find : $m(\angle EMA)$



3 [a] State two cases of the cyclic quadrilateral.

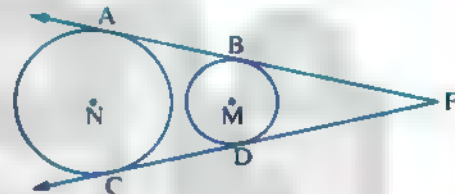
[b] In the opposite figure :

- $E \in \overline{AB}$, $E \notin \overline{AB}$
 $m(\widehat{AB}) = 110^\circ$
 $m(\angle CBE) = 85^\circ$
 Find : $m(\angle BDC)$



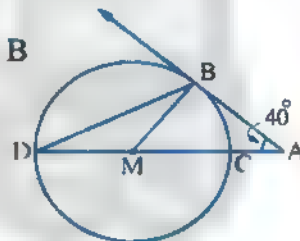
4 [a] In the opposite figure :

- \overline{AB} , \overline{CD} are common external tangents to the two circles M and N , $\overline{AB} \cap \overline{CD} = \{F\}$
 Prove that : $AB = CD$



[b] In the opposite figure :

- A is a point outside the circle M , \overline{AB} is a tangent to the circle at B
 \overline{AM} intersects the circle M at C and D respectively
 $m(\angle A) = 40^\circ$
 Find with proof : $m(\angle BDC)$

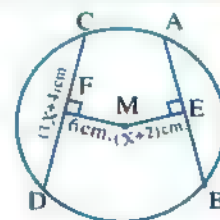


5 [a] In the opposite figure :

$$AB = CD$$

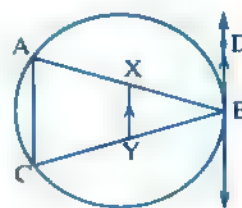
Find : 1 The value of x

2 The length of \overline{CD}



[b] In the opposite figure :

- ABC is a triangle inscribed in a circle
 \overline{BD} is a tangent to the circle at B , $X \in \overline{AB}$
 $Y \in \overline{CB}$ where $\overline{YX} \parallel \overline{BD}$
 Prove that : AXYC is a cyclic quadrilateral.



Geometry

20

Qena Governorate



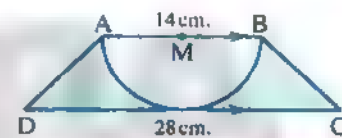
Answer the following questions : (Calculators are permitted)

1 Choose the correct answer :

- 1 The measure of the inscribed angle in a semicircle is °
 (a) 45 (b) 90 (c) 135 (d) 180
- 2 The perimeter of a rhombus is 12 cm. , then the length of its side = cm.
 (a) 3 (b) 4 (c) 6 (d) 8
- 3 If A and B are two points in the plane , $AB = 7$ cm. , then the length of the diameter of the smallest circle passing through the two points A and B equals cm.
 (a) 3 (b) 3.5 (c) 7 (d) 14

4 In the opposite figure :

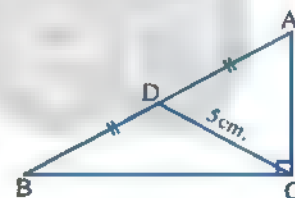
\overline{AB} is a diameter of the circle M , \overline{CD} is a tangent
 , $AB = 14$ cm. , $CD = 28$ cm.
 , then the area of the shaded part = cm^2



- (a) 70 (b) 147 (c) 170 (d) 224
- 5 It is possible to draw a circle passing through the vertices of a
 (a) rhombus. (b) rectangle. (c) trapezium. (d) parallelogram.

6 In the opposite figure :

$\triangle ABC$ is right-angled at C
 , \overline{CD} is a median , $CD = 5$ cm.
 , then $AB =$ cm.



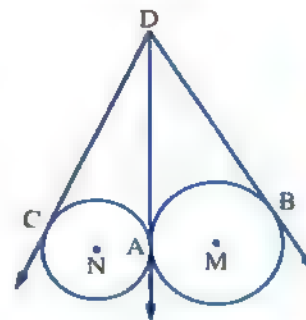
- (a) 4 (b) 6
 (c) 8 (d) 10

- 2 [a] Find the length of the arc and its measure , which is opposite to an inscribed angle of measure 45° in a circle the length of its radius is 7 cm.

[b] In the opposite figure :

M and N are two circles touching externally at A
 , \overline{DA} is a common tangent to the circles
 , \overline{DB} is a tangent to the circle M at B
 , \overline{DC} is a tangent to the circle N at C

Prove that : $DB = DC$



3 [a] In the opposite figure :

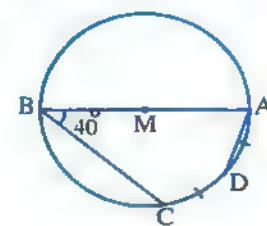
\overline{AB} is a diameter of the circle M

, D is the midpoint of \widehat{AC}

, $m(\angle ABC) = 40^\circ$

Find : 1 $m(\angle DAB)$

2 $m(\angle DCB)$



[b] In the opposite figure :

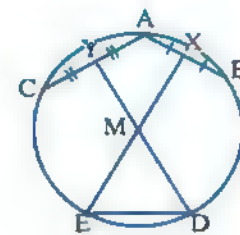
\overline{AB} , \overline{AC} are two chords in the circle M

, X and Y are the two midpoints of \overline{AB} and \overline{AC} respectively

, \overline{YM} and \overline{XM} intersect the circle at D and E

If $DE = r$ where r is the radius length of M

, find by proof : $m(\angle BAC)$



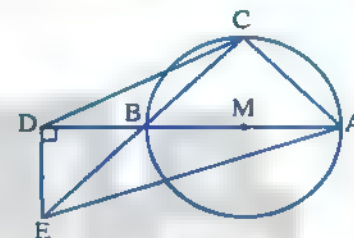
4 [a] In the opposite figure :

\overline{AB} is a diameter in the circle M

, $D \in \overline{AB}$, $D \notin \overline{AB}$, $\overline{DE} \perp \overline{AB}$

, $C \in \widehat{AB}$, $\overline{CB} \cap \overline{DE} = \{E\}$

Prove that : ACDE is a cyclic quadrilateral



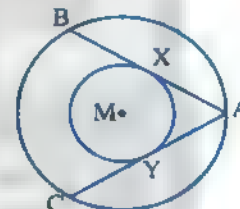
[b] In the opposite figure :

Two concentric circles of center M

, \overline{AB} and \overline{AC} are two chords in the greater

circle and tangents to the smaller circle at X and Y respectively.

Prove that : $AB = AC$



5 [a] In the opposite figure :

M and N are two intersecting circles at A and B

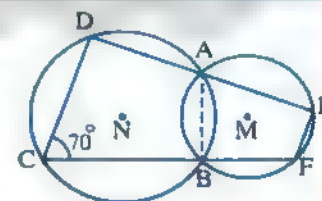
, \overline{AD} is drawn to intersect the circle M at E and

the circle N at D, \overline{AB} is drawn to intersect the circle M

at F and the circle N at C, $m(\angle BCD) = 70^\circ$

1 Find : $m(\angle EFB)$

2 Prove that : $\overline{CD} \parallel \overline{EF}$



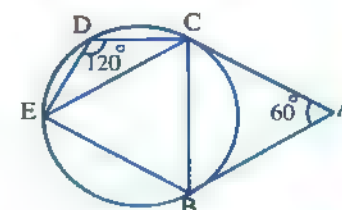
[b] In the opposite figure :

\overline{AB} and \overline{AC} are tangent-segments to the circle at B and C

, $m(\angle BAC) = 60^\circ$, $m(\angle CDE) = 120^\circ$

Prove that : 1 $\triangle BCE$ is an equilateral triangle.

2 $\overline{AC} \parallel \overline{BE}$



Geometry

21

Luxor Governorate



Answer the following questions :

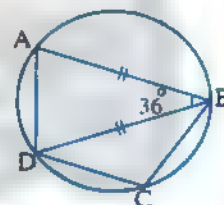
1 Choose the correct answer :

- 1 The number of axes of symmetry of the rectangle is
 (a) 1 (b) 2 (c) 3 (d) 4
- 2 If M, N are two circles whose radii lengths are r_1, r_2 and if $r_1 - r_2 < MN < r_1 + r_2$, then the two circles are
 (a) distant. (b) concentric. (c) intersecting. (d) touching.
- 3 The length of the median drawn from the vertex of the right angle in the right-angled triangle equals the length of the hypotenuse.
 (a) quarter (b) twice (c) half (d) three quarters
- 4 The length of the arc subtending a central angle of measure 60° in a circle whose circumference is 24 cm. equals cm.
 (a) 4 (b) 8 (c) 12 (d) 16
- 5 The measure of the exterior angle of the equilateral triangle is
 (a) 30 (b) 60 (c) 90 (d) 120

6 In the opposite figure :

$AB = BD$, $m(\angle ABD) = 36^\circ$
 , then $m(\angle C) = \dots\dots\dots^\circ$

- (a) 140 (b) 108
 (c) 70 (d) 54



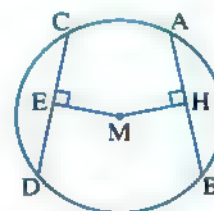
2 [a] In the opposite figure :

$AB = CD$, $\overline{MH} \perp \overline{AB}$, $\overline{ME} \perp \overline{CD}$

If $ME = 6$ cm. , $MH = (x + 2)$ cm.

and $CD = (3x + 4)$ cm.

, find : The value of x and the length of \overline{AB}

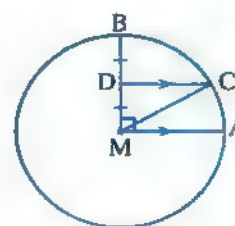


[b] In the opposite figure :

$\overline{AM} \parallel \overline{CD}$

, $MD = DB$, $m(\angle AMB) = 90^\circ$

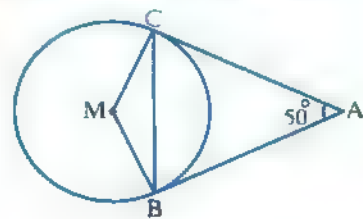
Find : $m(\widehat{AC})$



3 [a] In the opposite figure :

\overline{AB} , \overline{AC} are two tangent segments drawn to the circle from A at B, C respectively, $m(\angle A) = 50^\circ$

Find : $m(\angle ACB)$, $m(\angle BCM)$



[b] In the opposite figure :

$m(\widehat{AX}) = m(\widehat{AY})$

Prove that :

1 DBCH is a cyclic quadrilateral.

2 $m(\angle DHB) = m(\angle XAB)$

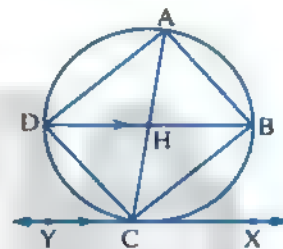
4 [a] Draw \overline{AB} of length 3 cm, then draw a circle passing by the two points A, B whose radius length is 2 cm. How many possible solutions are there ?

[b] In the opposite figure :

$\overline{BD} \parallel \overline{XY}$

Prove that : 1 \overline{AC} bisects $\angle BAD$

2 \overline{BC} is a tangent to the circle passing by the vertices of $\triangle ABH$



5 [a] In the opposite figure :

ABCD is a parallelogram

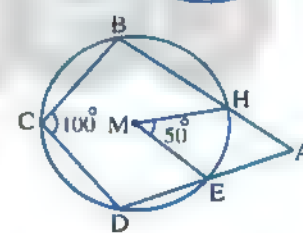
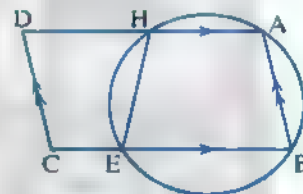
Prove that : HDCE is a cyclic quadrilateral

[b] In the opposite figure :

$m(\angle M) = 50^\circ$

$m(\angle C) = 100^\circ$

Find : $m(\angle A)$



22

Aswan Governorate



Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

1 The measure of the inscribed angle drawn in a semicircle equals

(a) 45°

(b) 180°

(c) 120°

(d) 90°

Geometry

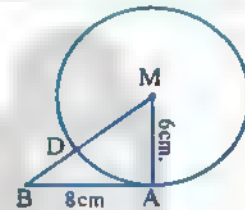
- 2 The number of symmetry axes of the isosceles triangle is
 (a) zero (b) 1 (c) 2 (d) 3
- 3 The surface of the circle $M \cap$ the surface of the circle $N = \{A\}$ and the radius length of one of them is 3 cm. and $MN = 8$ cm. , then the radius length of the other circle equals cm.
 (a) 5 (b) 6 (c) 11 (d) 16
- 4 The measure of the exterior angle of the equilateral triangle equals
 (a) 30° (b) 60° (c) 120° (d) 180°
- 5 The line segment joining the two midpoints of two sides of the triangle is the third side.
 (a) perpendicular to (b) parallel to (c) bisecting (d) equal to
- 6 If ABCD is a cyclic quadrilateral , then $m(\angle A) + m(\angle C) - 80^\circ = \dots\dots\dots$
 (a) 60° (b) 80° (c) 100° (d) 120°

2 [a] In the opposite figure :

\overline{AB} is a tangent to the circle M at A

, $MA = 6$ cm. , $AB = 8$ cm.

Find : The length of \overline{BD}



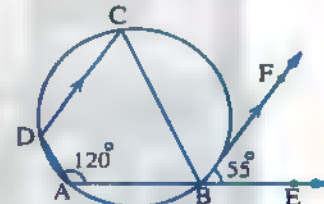
[b] In the opposite figure :

ABCD is a cyclic quadrilateral

, $\overline{BF} \parallel \overline{DC}$, $m(\angle BAD) = 120^\circ$

, $m(\angle EBF) = 55^\circ$

Find : $m(\angle BCD)$, $m(\angle ADC)$



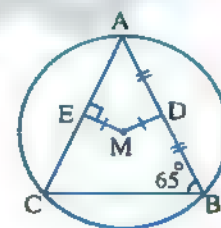
3 [a] In the opposite figure :

In the circle M

, $MD = ME$, D is the midpoint of \overline{AB}

, $\overline{ME} \perp \overline{AC}$, $m(\angle ABC) = 65^\circ$

Find : $m(\angle BAC)$

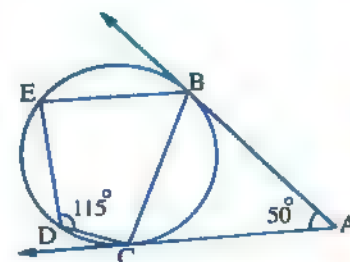


[b] In the opposite figure :

\overline{AB} , \overline{AC} are two tangents to the circle at B and C

, $m(\angle A) = 50^\circ$, $m(\angle CDE) = 115^\circ$

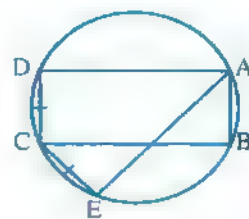
Prove that : \overline{BC} bisects $\angle ABE$



4 [a] In the opposite figure :

ABCD is a rectangle inscribed in a circle , the chord \overline{CE} is drawn where $CE = CD$

Prove that : $AE = BC$

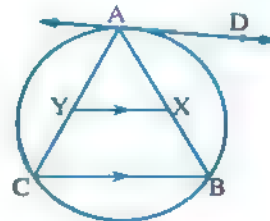


[b] In the opposite figure :

ABC is a triangle inscribed in a circle
 \overline{AD} is a tangent to the circle at A
 $X \in \overline{AB}$, $Y \in \overline{AC}$, $\overline{XY} \parallel \overline{BC}$

Prove that :

\overline{AD} is a tangent to the circle passing through the vertices of $\triangle AXY$



5 [a] In the opposite figure :

\overline{AC} , \overline{DB} are two parallel chords in the circle M
 $m(\angle AMB) = 140^\circ$

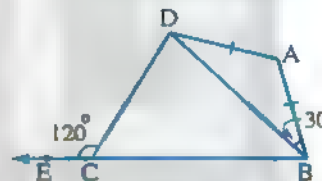
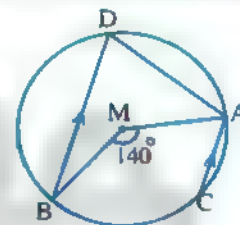
Find : $m(\angle D)$, $m(\angle DAC)$

[b] In the opposite figure :

$AB = AD$, $m(\angle ABD) = 30^\circ$

$m(\angle DCE) = 120^\circ$

Prove that : ABCD is a cyclic quadrilateral.



23

New Valley Governorate



Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

- [1] If two polygons are similar and the ratio between the lengths of two corresponding sides is 1 : 3 and the perimeter of the smaller polygon is 15 cm. , then the perimeter of the greater polygon is cm.

(a) 30 (b) 45 (c) 60 (d) 75

- [2] The inscribed angle drawn in a semicircle is

(a) acute. (b) obtuse. (c) straight. (d) right.

- [3] ABC is a right-angled triangle at B , $\overline{BD} \perp \overline{AC}$, then the projection of \overline{BD} on \overline{AC} is

(a) A (b) B (c) C (d) D

Geometry

- 4 A tangent to a circle of diameter length 6 cm. is at a distance of cm. from its center.

(a) 6 (b) 12 (c) 3 (d) 2

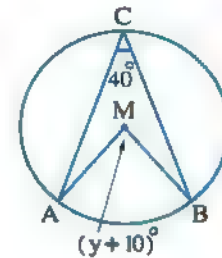
- 5 In the opposite figure :

If $m(\angle AMB) = (y + 10)^\circ$

, $m(\angle C) = 40^\circ$

, then $y = \dots\dots\dots$

(a) 70° (b) 80°
(c) 100° (d) 180°



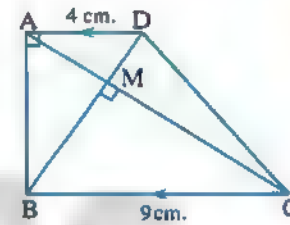
- 6 In the opposite figure :

$\overline{AD} \parallel \overline{BC}$, $m(\angle BAD) = m(\angle BMC) = 90^\circ$

, $AD = 4$ cm. , $BC = 9$ cm.

, then the area of the trapezium ABCD = cm^2

(a) 26 (b) 39
(c) 52 (d) 65



- 2 [a] In the opposite figure :

$m(\angle ABE) = 100^\circ$

, $m(\angle CAD) = 40^\circ$

Prove that : $m(\widehat{CD}) = m(\widehat{AD})$

- [b] In the opposite figure :

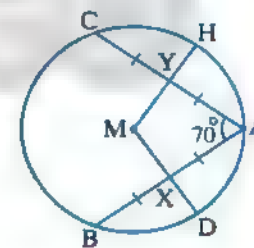
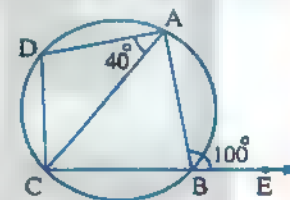
\overline{AB} and \overline{AC} are two chords equal

in length in the circle M , X is the midpoint of \overline{AB}

, Y is the midpoint of \overline{AC} , $m(\angle CAB) = 70^\circ$

1 Calculate : $m(\angle DMH)$

2 Prove that : $XD = YH$



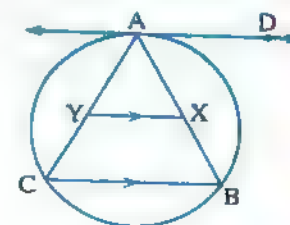
- 3 [a] In the opposite figure :

ABC is a triangle inscribed in a circle

, \overline{AD} is a tangent to the circle at A

, $X \in \overline{AB}$, $Y \in \overline{AC}$ where $\overline{XY} \parallel \overline{BC}$

Prove that : \overline{AD} is a tangent to the circle passing through the points A , X and Y

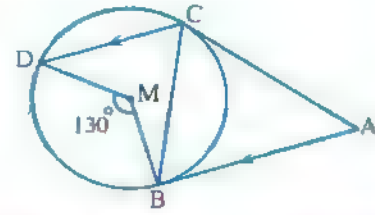


[b] In the opposite figure :

\overline{AB} and \overline{AC} are two tangent-segments to the circle M
 $\overline{AB} \parallel \overline{CD}$, $m(\angle BMD) = 130^\circ$

1 Prove that : \overline{CB} bisects $\angle ACD$

2 Find : $m(\angle A)$



4 [a] In the opposite figure :

ABC is an inscribed triangle inside a circle

$\overline{DE} \parallel \overline{BC}$

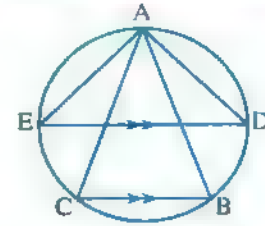
Prove that : $m(\angle DAC) = m(\angle BAE)$

[b] ABC is a triangle inscribed in a circle , $X \in \widehat{AB}$, $Y \in \widehat{AC}$

where $m(\widehat{AX}) = m(\widehat{AY})$, $\overline{CX} \cap \overline{AB} = \{D\}$, $\overline{BY} \cap \overline{AC} = \{E\}$

Prove that : 1 BCED is a cyclic quadrilateral.

2 $m(\angle DEB) = m(\angle XAB)$



5 [a] State two cases of the cyclic quadrilateral.

[b] In the opposite figure :

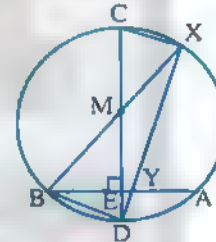
\overline{AB} is a chord in the circle M and \overline{CD} is the

perpendicular diameter on \overline{AB} and intersects it at E

\overline{BM} intersects the circle at X and $\overline{XD} \cap \overline{AB} = \{Y\}$

Prove that : 1 Xyec is a cyclic quadrilateral.

2 $m(\angle DYB) = m(\angle DBX)$



24 South Sinai Governorate



Answer the following questions :

1 Choose the correct answer from those given :

1 The measure of the inscribed angle drawn in a semicircle equals

- (a) 90° (b) 45° (c) 180° (d) 120°

2 A rhombus whose two diagonals lengths are 6 cm. , 8 cm. , then its area is cm^2

- (a) 14 (b) 24 (c) 48 (d) 12

3 If ABCD is a cyclic quadrilateral , then $m(\angle A) + m(\angle C) - 90^\circ = \dots\dots\dots$

- (a) 180° (b) 100° (c) 90° (d) 120°

Geometry

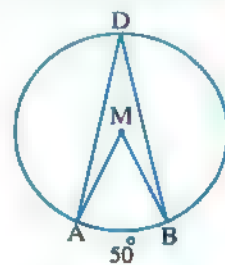
- 4 In the triangle ABC , where $(AB)^2 + (BC)^2 < (AC)^2$, then $\angle B$ is
 (a) right. (b) acute. (c) straight. (d) obtuse.
- 5 The sum of measures of the interior angles of the triangle equals
 (a) 180° (b) 90° (c) 100° (d) 360°
- 6 The number of axes of symmery of the circle is
 (a) zero (b) an infinite number
 (c) 2 (d) 3

2 [a] In the opposite figure :

$$m(\widehat{AB}) = 50^\circ$$

Find : 1 $m(\angle D)$

2 $m(\angle AMB)$

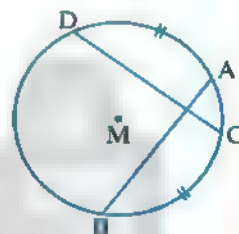


[b] In the opposite figure :

\overline{AB} and \overline{CD} are two chords in the circle M

$$m(\widehat{AD}) = m(\widehat{BC})$$

Prove that : $AB = CD$



- 3 [a] If the radius length of the circle M is 5 cm. and the radius length of the circle N is 3 cm. , $MN = 8$ cm. , show the position of the two circles.

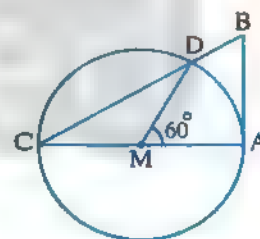
[b] In the opposite figure :

\overline{AB} is a tangent-segment to the circle M

\overline{AC} is a diameter of it and $m(\angle AMD) = 60^\circ$

1 Find : $m(\angle ABC)$

2 Prove that : $AB = \frac{1}{2} BC$



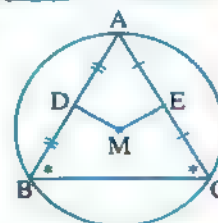
4 [a] In the opposite figure :

$$m(\angle B) = m(\angle C)$$

\overline{D} is the midpoint of \overline{AB}

\overline{E} is the midpoint of \overline{AC}

Prove that : $MD = ME$

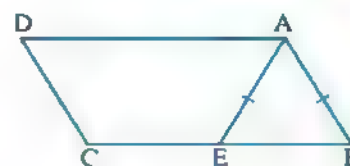


[b] In the opposite figure :

ABCD is a parallelogram

and $E \in \overline{BC}$, such that : $AB = AE$

Prove that : The figure AECD is a cyclic quadrilateral.



Final Examinations

5 [a] In the opposite figure :

\overline{AB} and \overline{AC} are two tangents to the circle at B and C

, $m(\angle A) = 50^\circ$, $m(\angle EDC) = 115^\circ$

Prove that : 1 \overline{BC} bisects $\angle ABE$

2 $CB = CE$

[b] In the opposite figure :

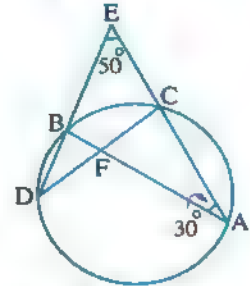
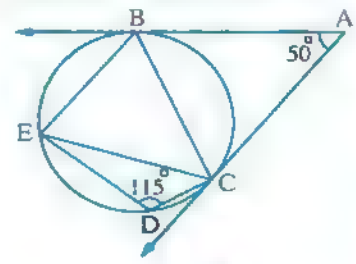
$\overline{AB} \cap \overline{CD} = \{F\}$, $\overline{AC} \cap \overline{DB} = \{E\}$

, $m(\angle A) = 30^\circ$

, $m(\angle E) = 50^\circ$

Find : 1 $m(\widehat{AD})$

2 $m(\angle AFD)$



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North Sinai Governorate

Answer the following questions :

1 Choose the correct answer from those given :

1 If the surface of the circle $M \cap$ the surface of the circle $N = \{A\}$, then M, N are

(a) distant. (b) concentric. (c) touching externally. (d) intersecting.

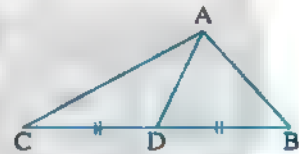
2 In the opposite figure :

\overline{AD} is a median in the triangle ABC

, the area of the triangle ABD = 20 cm²

, then the area of the triangle ACD = cm²

(a) 20 (b) 40 (c) 60 (d) 80

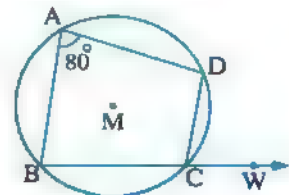


3 In the opposite figure :

If $m(\angle BAD) = 80^\circ$

, then $m(\angle DCW) = \dots\dots\dots^\circ$

(a) 30 (b) 80
(c) 60 (d) 120



4 The area of the square whose diagonal length is 4 cm. equals cm²

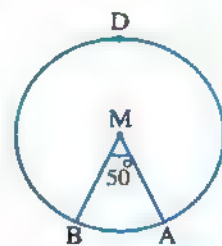
(a) 4 (b) 8 (c) 16 (d) 16 π

5 In the opposite figure :

$m(\angle AMB) = 50^\circ$

, then $m(\widehat{ADB}) = \dots\dots\dots^\circ$

(a) 50 (b) 100
(c) 310 (d) 350



Geometry

- 6 A triangle having one symmetry line and its side lengths are 8 , 4 , X cm.
then X =

(a) 2 (b) 4 (c) 8 (d) 12

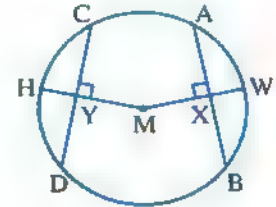
- 2 [a] In the opposite figure :

If $AB = CD$

, $\overline{MW} \perp \overline{AB}$

, $\overline{MH} \perp \overline{CD}$

Prove that : $WX = HY$



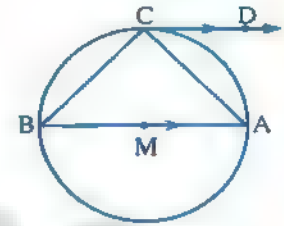
- [b] In the opposite figure :

\overline{CD} is a tangent to the circle M at C

, $\overline{CD} \parallel \overline{BA}$ and $M \in \overline{AB}$

1 Prove that : $AC = BC$

2 Find : $m(\angle B)$



- 3 [a] State two cases in which the figure is a cyclic quadrilateral.

- [b] In the opposite figure :

\overline{BC} is a diameter in the circle M

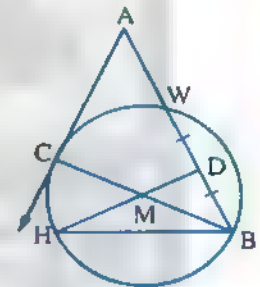
, \overline{AC} is a tangent to the circle M at C

, D is the midpoint of \overline{BW}

Prove that :

1 The figure ADMC is a cyclic quadrilateral.

2 $m(\angle CBH) = \frac{1}{2} m(\angle A)$



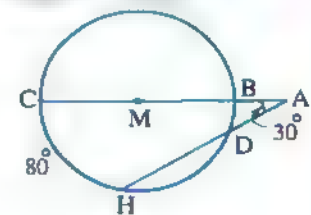
- 4 [a] In the opposite figure :

\overline{BC} is a diameter in the circle M

, $\overline{CA} \cap \overline{HA} = \{A\}$, $m(\angle A) = 30^\circ$

and $m(\widehat{CH}) = 80^\circ$

Find : $m(\widehat{DH})$

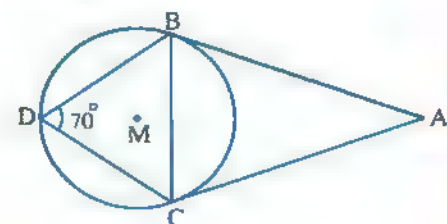


- [b] In the opposite figure :

\overline{AB} , \overline{AC} are two tangent-segments
to the circle at B and C

and $m(\angle BDC) = 70^\circ$

Find : $m(\angle BAC)$

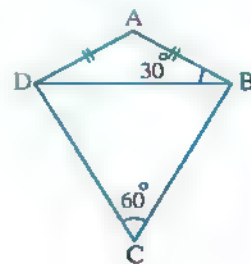


5 [a] In the opposite figure :

$AB = AD$, $m(\angle ABD) = 30^\circ$

and $m(\angle C) = 60^\circ$

Prove that : ABCD is a cyclic quadrilateral.



[b] By using geometric instruments , draw $\triangle ABC$ where
 $AB = 3$ cm. , $BC = 4$ cm. , $AC = 5$ cm. , then draw a circle passing through the
 vertices of $\triangle ABC$

How many circles are there ?

26

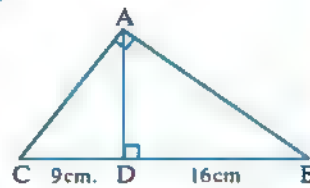
Red Sea Governorate



Answer the following questions :

1 Choose the correct answer from the given answers :

- 1 The angle of tangency is included between
 - (a) two chords.
 - (b) two tangents.
 - (c) a chord and a tangent.
 - (d) a chord and a diameter.
- 2 The number of symmetry axes of the semicircle is
 - (a) zero
 - (b) 1
 - (c) 3
 - (d) an infinite number.
- 3 A circle of circumference 6π cm. and a straight line L is at 3 cm. distant from its centre , then L is
 - (a) a tangent.
 - (b) a secant.
 - (c) outside the circle.
 - (d) a diameter of the circle.
- 4 The inscribed angle in a semicircle is angle.
 - (a) an acute
 - (b) an obtuse
 - (c) a straight
 - (d) a right
- 5 The radius length of the circle whose centre is the point of origin and passes through $(-3, 4)$ equals length unit.
 - (a) 3
 - (b) 4
 - (c) 5
 - (d) 7
- 6 In the opposite figure :
 ABC is a right-angled triangle at A
 $\overline{AD} \perp \overline{BC}$, $BD = 16$ cm.
 $CD = 9$ cm. , then $AB =$ cm.
 - (a) 5
 - (b) 7
 - (c) 20
 - (d) 25

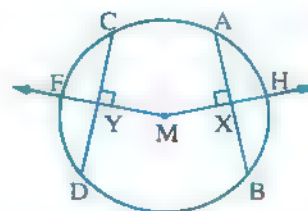


Geometry

2 [a] In the opposite figure :

\overline{AB} and \overline{CD} are two chords equal in length in the circle M
 $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$

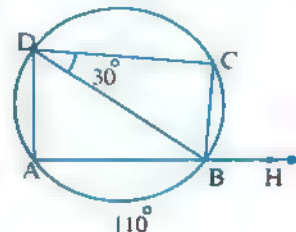
Prove that : $HX = FY$



[b] In the opposite figure :

$H \in \overline{AB}$, $m(\widehat{AB}) = 110^\circ$
 $m(\angle CDB) = 30^\circ$

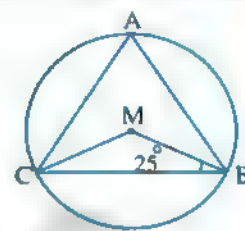
Find : $m(\angle HBC)$



3 [a] In the opposite figure :

ABC is a triangle drawn in the circle M
 $m(\angle MBC) = 25^\circ$

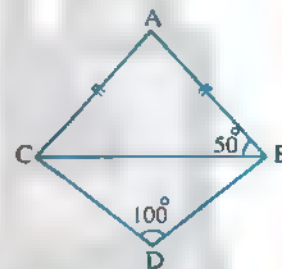
Find : $m(\angle BAC)$



[b] In the opposite figure :

$AB = AC$, $m(\angle D) = 100^\circ$
 $m(\angle ABC) = 50^\circ$

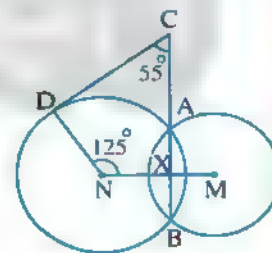
Prove that : ABDC is a cyclic quadrilateral.



4 [a] In the opposite figure :

M and N are two intersecting circles at A and B
 $C \in \overline{BA}$, $D \in$ the circle N , $m(\angle MND) = 125^\circ$
 $m(\angle C) = 55^\circ$

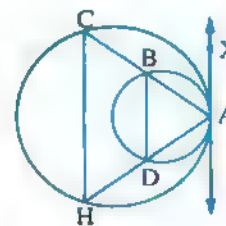
Prove that : \overline{CD} is a tangent to the circle N at D



[b] In the opposite figure :

\overline{AX} is a common tangent for the two circles touching internally at A

Prove that : $\overline{BD} \parallel \overline{CH}$



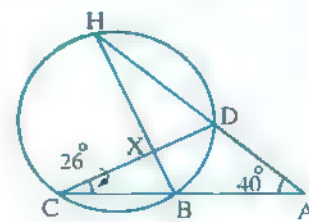
5 [a] In the opposite figure :

$$\overline{CB} \cap \overline{HD} = \{A\}, m(\angle A) = 40^\circ$$

$$, \overline{CD} \cap \overline{BH} = \{X\}$$

$$, m(\angle DCB) = 26^\circ$$

Find : $m(\widehat{CH})$, $m(\angle HXC)$



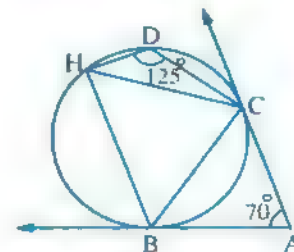
[b] In the opposite figure :

\overline{AB} and \overline{AC} are two tangents to the circle at B and C

$$, m(\angle A) = 70^\circ$$

$$, m(\angle CDH) = 125^\circ$$

Prove that : $CB = CH$



27

Matrouh Governorate



Answer the following questions :

1 Choose the correct answer from those given :

1 In the cyclic quadrilateral , each two opposite angles are

(a) equal in measure.

(b) complementary.

(c) supplementary.

(d) alternate.

2 A square is of perimeter 20 cm. , then its area equals

(a) 50 cm²

(b) 50 cm.

(c) 25 cm²

(d) 25 cm.

3 ΔABC is right-angled at B , if $BC = 8$ cm. , $AB = 6$ cm. , then $\sin C = \dots\dots\dots$

(a) $\frac{3}{4}$

(b) $\frac{4}{3}$

(c) $\frac{5}{3}$

(d) 0.6

4 The ratio between the measure of the central angle and the measure of the inscribed angle subtended by the same arc equals

(a) 1 : 2

(b) 2 : 1

(c) 1 : 3

(d) 1 : 4

5 The measure of the angle of the regular pentagon is equal to

(a) 72°

(b) 180°

(c) 108°

(d) 120°

6 A chord with length 8 cm. in a circle with circumference 10π cm. , then it is distant from its center by

(a) 2 cm.

(b) 3 cm.

(c) 4 cm.

(d) 5 cm.

2 [a] \overline{AB} and \overline{AC} are two chords equal in length in a circle M , X and Y are the midpoints of \overline{AB} and \overline{AC} respectively, $m(\angle MXY) = 30^\circ$

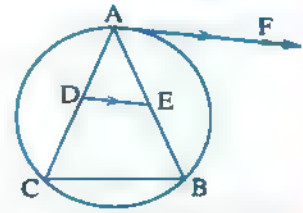
Prove that : ΔMXY is an isosceles triangle.

Geometry

[b] In the opposite figure :

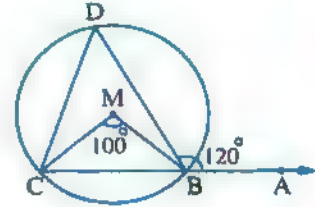
\overrightarrow{AF} is a tangent to the circle at A
 $\overrightarrow{AF} \parallel \overrightarrow{DE}$

Prove that : DEBC is a cyclic quadrilateral.



3 [a] In the opposite figure :

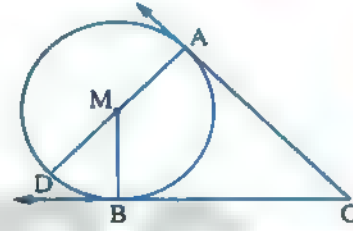
A circle of center M

 $m(\angle BMC) = 100^\circ$ $m(\angle ABD) = 120^\circ$ Find : $m(\angle DCB)$ 

[b] In the opposite figure :

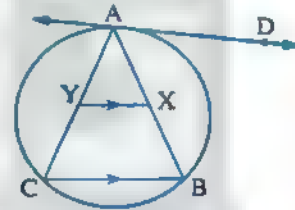
 \overrightarrow{AD} is a diameter in the circle M

\overrightarrow{CA} and \overrightarrow{CB} are two tangents to the circle M
 , touching it at A and B respectively.

Prove that : $m(\angle DMB) = m(\angle ACB)$ 

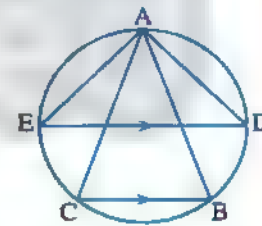
4 [a] In the opposite figure :

ABC is a triangle inscribed in a circle

 \overrightarrow{AD} is a tangent to the circle at A $X \in \overrightarrow{AB}$, $Y \in \overrightarrow{AC}$ where $\overrightarrow{XY} \parallel \overrightarrow{BC}$ Prove that : \overrightarrow{AD} is a tangent to the circle passing through the points A , X and Y

[b] In the opposite figure :

ABC is an inscribed triangle inside a circle

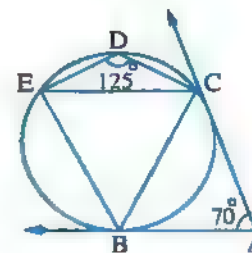
 $\overrightarrow{DE} \parallel \overrightarrow{BC}$ Prove that : $m(\angle DAC) = m(\angle BAE)$ 

5 [a] Prove that : In the same circle , the measures of all inscribed angles subtended by the same arc are equal.

[b] In the opposite figure :

 \overrightarrow{AB} , \overrightarrow{AC} are two tangents to the circle at B , C $m(\angle A) = 70^\circ$, $m(\angle CDE) = 125^\circ$

Prove that : 1 CB = CE

2 $\overrightarrow{AC} \parallel \overrightarrow{BE}$ 

Geometry

Answers of school book examinations in geometry

Model 1

1

1 d 2 a 3 d 4 a 5 b 6 a

2

[a] supplementary, theoretical.

[b] $\because \overline{XY} \parallel \overline{BD}$, \overline{AB} is a transversal $\therefore m(\angle DBX) = m(\angle BXY)$ (alternate angles) (1) $\because m(\angle C)$ (inscribed) $= m(\angle ABD)$ (tangency) (2)From (1) and (2): $\therefore m(\angle C) = m(\angle BXY)$ \therefore AXYC is a cyclic quadrilateral. (Q.E.D.)

3

[a] $\because \overline{AB}$, \overline{AC} are two tangents to the smaller circle $\therefore AB = AC \quad \therefore 2x - 3 = 15$ $\therefore 2x = 18 \quad \therefore x = 9$ $\because \overline{AB}$, \overline{AD} are two tangents to the greater circle $\therefore AB = AD \quad \therefore y - 2 = 15 \quad \therefore y = 17$ [b] $\because m(\angle BDC) = m(\angle BAC)$ (two inscribed angles subtended by \widehat{BC}) $\therefore m(\angle BDC) = 30^\circ$ $\therefore m(\widehat{BC}) = 2m(\angle BAC) = 60^\circ$ $\because \overline{AB}$ is a diameter in the circle M $\therefore m(\widehat{AB}) = 180^\circ$ $\therefore m(\widehat{AC}) = 180^\circ - 60^\circ = 120^\circ$ \because D is the midpoint of \widehat{AC} $\therefore m(\widehat{AD}) = \frac{120^\circ}{2} = 60^\circ$ (First req.) $\therefore m(\angle ACD) = \frac{1}{2} m(\widehat{AD}) = \frac{1}{2} \times 60^\circ = 30^\circ$ $\therefore m(\angle CAB) = m(\angle ACD)$ but they are alternate $\therefore \overline{AB} \parallel \overline{DC}$ (Second req.)

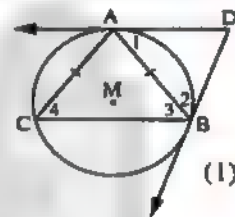
4

[a] \because X is the midpoint of \overline{AB} $\therefore \overline{MX} \perp \overline{AB} \quad \therefore m(\angle MXA) = 90^\circ$ \because Y is the midpoint of \overline{AC} $\therefore \overline{MY} \perp \overline{AC} \quad \therefore m(\angle MYA) = 90^\circ$

From the quadrilateral AXMY:

 $\therefore m(\angle DMH) = 360^\circ - (90^\circ + 90^\circ + 70^\circ) = 110^\circ$
(First req.) $\because AB = AC$, $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$ $\therefore MX = MY \quad \therefore MD = MH = r$ By subtracting: $\therefore XD = YH$ (Second req.)[b] $\because m(\angle A) = \frac{1}{2} [m(\widehat{HC}) - m(\widehat{BD})]$ $\therefore 30^\circ = \frac{1}{2} [120^\circ - m(\widehat{BD})]$ $\therefore 60^\circ = 120^\circ - m(\widehat{BD}) \quad \therefore m(\widehat{BD}) = 120^\circ - 60^\circ$ $\therefore m(\widehat{BD}) = 60^\circ$ (First req.) $\therefore m(\widehat{BC}) = m(\widehat{DH}) \quad \therefore BC = DH$ by adding $m(\widehat{BD})$ to both sides $\therefore m(\widehat{CD}) = m(\widehat{HB}) \quad \therefore m(\angle C) = m(\angle H)$ In $\triangle ACH$: $\therefore AC = AH$, $\therefore BC = DH$ By subtracting: $\therefore AB = AD$ (Second req.)

5

[a] $\because \overline{DA}$ and \overline{DB} are two tangent-segments to the circle M at A and B $\therefore DA = DB$ $\therefore m(\angle 1) = m(\angle 2)$ $\therefore m(\angle D) = 180^\circ - 2m(\angle 1)$ In $\triangle ABC$: $\because AB = AC$ $\therefore m(\angle 3) = m(\angle 4)$ $\therefore m(\angle BAC) = 180^\circ - 2m(\angle 4)$ (2) $\because \overline{AD}$ is a tangent-segment to the circle $\therefore m(\angle 4)$ (inscribed) $= m(\angle 1)$ (tangency) (3)From (1), (2) and (3): $\therefore m(\angle BAC) = m(\angle D)$ $\therefore \overline{AC}$ is a tangent to the circle passing through the vertices of the $\triangle ABD$ (Q.E.D.)[b] In $\triangle AMB$: $\because AM = BM = r$ $\therefore m(\angle MBA) = m(\angle MAB) = 20^\circ$ \because C is the midpoint of \overline{AB} $\therefore \overline{MC} \perp \overline{AB} \quad \therefore m(\angle MCB) = 90^\circ$ In $\triangle BCM$: $\therefore m(\angle BMC) = 180^\circ - (90^\circ + 20^\circ) = 70^\circ$ $\therefore m(\angle BHD) = \frac{1}{2} m(\angle BMD)$ (inscribed and central angles subtended by \widehat{BD}) $\therefore m(\angle BHD) = \frac{1}{2} \times 70^\circ = 35^\circ$ (First req.)In $\triangle AMB$: $\because AM = BM = r$ $\therefore m(\angle MAB) = m(\angle MBA) = 20^\circ$ $\therefore m(\angle AMB) = 180^\circ - (20^\circ + 20^\circ) = 140^\circ$ $\therefore m(\widehat{ADB}) = m(\angle AMB) = 140^\circ$ (Second req.)

Model 2

1

1 b

2 d

3 b

4 c

5 d

6 b

2

[a] $\because AB = AC$ $\therefore MD \perp AB, ME \perp AC$ $\therefore MD = ME$ $\therefore MX = MY = r$ $\therefore DX = EY$

(Q.E.D.)

[b] In $\triangle ABD$: $\because AB = AD$ $\therefore m(\angle ABD) = m(\angle ADB) = 30^\circ$ $\therefore m(\angle A) = 180^\circ - 2 \times 30^\circ = 120^\circ$ $\therefore m(\angle A) + m(\angle C) = 120^\circ + 60^\circ = 180^\circ$ $\therefore ABCD$ is a cyclic quadrilateral. (Q.E.D.)

3

[a] State by yourself.

[b] $\because E$ is the midpoint of \widehat{BF} $\therefore m(\widehat{FE}) = m(\widehat{BE})$ $\therefore m(\angle FAE) = m(\angle BAE)$ $\therefore m(\angle CBE)$ (tangency) $= m(\angle BAE)$

(inscribed)

 $\therefore m(\angle DAC) = m(\angle DBC)$ and they are drawn on \widehat{DC} and on one side of it $\therefore ABCD$ is a cyclic quadrilateral.

4

[a] $\because \overline{AD}, \overline{AF}$ are two tangent-segments to the circle $\therefore AD = AF = 5$ cm. $\because \overline{BD}, \overline{BE}$ are two tangent-segments to the circle $\therefore BD = BE = 4$ cm. $\because \overline{CE}, \overline{CF}$ are two tangent-segments to the circle $\therefore CE = CF = 3$ cm. \therefore The perimeter of $\triangle ABC = 5 + 5 + 4 + 4 + 3 + 3 = 24$ cm. (The req.)[b] $\because \overline{AF} \parallel \overline{DE}, \overline{AB}$ is a transversal $\therefore m(\angle AED) = m(\angle EAF)$ (alternate angles) $\therefore m(\angle C)$ (inscribed) $= m(\angle BAF)$ (tangency) $\therefore m(\angle C) = m(\angle AED)$ $\therefore DEBC$ is a cyclic quadrilateral. (Q.E.D.)

5

 $\because BCDE$ is a cyclic quadrilateral $\therefore m(\angle CBE) + m(\angle D) = 180^\circ$ $\therefore m(\angle CBE) = 180^\circ - 125^\circ = 55^\circ$ $\because \overline{AB}, \overline{AC}$ are two tangents to the circle $\therefore AB = AC$ \therefore In $\triangle ABC$: $m(\angle ACB) = m(\angle ABC)$

$$= \frac{180^\circ - 70^\circ}{2} = 55^\circ$$

 $\therefore m(\angle CBE) = m(\angle ACB) = 55^\circ$

and they are alternate angles

 $\therefore \overline{AC} \parallel \overline{BE}$ $\therefore m(\angle BEC)$ (inscribed) $= m(\angle ACB)$ (tangency) $= 55^\circ$ $\therefore m(\angle CBE) = m(\angle BEC) = 55^\circ$ \therefore In $\triangle CBE$: $CB = CE$

Model examination for the merge students

1

1 diameter

2 perpendicular to this chord

3 equal

4 3

5 infinite

6 180°

2

1 a

2 a

3 d

4 c

5 d

6 c

3

1 x

2 ✓

3 x

4 ✓

5 x

6 x

4

1 90° 2 130° 3 40°

4 5

5 30°

6 2 : 1

Geometry

Answers of governorates' examinations of geometry

1 Cairo

1

1 c 2 b 3 a 4 a 5 c 6 d

2

[a] Mention by yourself.

[b] $\because \overline{AB}$ is a diameter in the circle M

$$\therefore m(\angle ACB) = 90^\circ \quad (1) \text{ (First req.)}$$

$$\because \overline{DE} \perp \overline{AD}$$

$$\therefore m(\angle ADE) = 90^\circ \quad (2)$$

From (1) and (2):

$$\therefore m(\angle ADE) = m(\angle ACE)$$

but they are drawn on \overline{AE} and on one side of it

\therefore The figure ACDE is a cyclic quadrilateral.
(Second req.)

3

[a] The measure of the arc = $\frac{1}{3} \times 360^\circ = 120^\circ$
(The req.)

[b] $\because m(\angle BAC) = \frac{1}{2} m(\angle BMC)$
(inscribed and central angles subtended the same arc \widehat{BC})

$$\therefore m(\angle BAC) = \frac{1}{2} \times 80^\circ = 40^\circ$$

$$\because AB = AC$$

$$\therefore m(\angle ABC) = m(\angle ACB) = \frac{180^\circ - 40^\circ}{2} = 70^\circ \quad (\text{First req.})$$

$$\therefore m(\widehat{BC}) = m(\angle M) = 80^\circ$$

$$\therefore m(\widehat{BC} \text{ the major}) = 360^\circ - 80^\circ = 280^\circ \quad (\text{Second req.})$$

4

[a] $\because \overline{MD} \perp \overline{AB}, \overline{ME} \perp \overline{CB}$

\because The sum of measures of the interior angles of the quadrilateral BDME = 360°

$$\therefore m(\angle DME) = 360^\circ - (70^\circ + 90^\circ + 90^\circ) = 110^\circ \quad (\text{First req.})$$

$$\therefore MD = ME, \overline{MD} \perp \overline{AB}, \overline{ME} \perp \overline{CB}$$

$$\therefore AB = CB \quad (\text{Second req.})$$

[b] $\because \overline{AB}, \overline{AC}$ are two tangents.

$$\therefore AB = AC$$

$$\therefore m(\angle ABC) = m(\angle ACB) \quad (1)$$

$\because \overline{BD} \parallel \overline{AC}, \overline{BC}$ is a transversal.

$$\therefore m(\angle DBC) = m(\angle ACB) \text{ (alternate angles)} \quad (2)$$

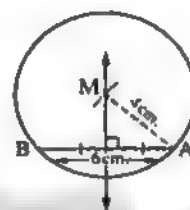
From (1) and (2):

$$\therefore m(\angle ABC) = m(\angle DBC)$$

$$\therefore \overline{BC} \text{ bisects } \angle ABD \quad (\text{Q.E.D.})$$

5

[a]



\therefore The radius length of the smallest circle = 3 cm.

[b] $\because \overline{AD}$ is a tangent to the circle at A

$$\therefore m(\angle ABC) \text{ (inscribed)} = m(\angle CAD) \text{ (tangency)} = 50^\circ$$

$$\because AC = BC$$

$$\therefore m(\angle BAC) = m(\angle ABC) = 50^\circ$$

$$\therefore m(\angle BEC) = m(\angle BAC) = 50^\circ$$

(two inscribed angles subtended by \widehat{BC})
(First req.)

$$\because m(\angle BEC) = m(\angle ABC) = 50^\circ$$

$\therefore \overline{BC}$ is a tangent to the circle passing through the vertices of $\triangle BEO$
(Second req.)

2 Giza

1

1 d 2 c 3 b 4 b 5 c 6 d

2

[a] $\because m(\angle A) = \frac{1}{2} m(\angle BMD) = \frac{1}{2} \times 150^\circ = 75^\circ$
(inscribed and central angles subtended by \widehat{BD})

$\because ABCD$ is a cyclic quadrilateral.

$$\therefore m(\angle C) = 180^\circ - 75^\circ = 105^\circ \quad (\text{The req.})$$

Answers of Final Examinations

[b] In $\triangle ABC$ $\therefore m(\angle B) = m(\angle C)$

$$\therefore AB = AC$$

$\therefore X$ is the midpoint of \overline{AB}

$$\therefore \overline{MX} \perp \overline{AB} \quad \therefore \overline{MY} \perp \overline{AC}$$

$$\therefore MX = MY$$

(Q.E.D.)

3

[a] Construction :

Draw $\overline{MX} \perp \overline{BD}$

$\therefore \overline{NY} \perp \overline{BE}$

Proof: $\therefore \overline{BD} \parallel \overline{MN}$

$\therefore \overline{MX} \perp \overline{BD}, \overline{NY} \perp \overline{BE}$

$$\therefore \overline{MX} \parallel \overline{NY}$$

\therefore The figure $MXYN$ is a rectangle

$\therefore X$ is midpoint of \overline{BD}

$\therefore Y$ is midpoint of \overline{BE}

$$\therefore DE = 2XY \quad \therefore XY = MN$$

$$\therefore DE = 2MN$$

(Q.E.D.)

[b] $\therefore \overline{AB}$ is a tangent to the circle M

$$\therefore \overline{MA} \perp \overline{AB} \quad \therefore m(\angle MAB) = 90^\circ$$

$$\text{In } \triangle MAB: \therefore m(\angle ABM) = 30^\circ, m(\angle MAB) = 90^\circ$$

$$\therefore BM = 2AM = 2 \times 8 = 16 \text{ cm}$$

$$\therefore (AB)^2 = (BM)^2 - (MA)^2 = (16)^2 - (8)^2 = 192$$

$$\therefore AB = 8\sqrt{3} \text{ cm.} \quad (\text{First req.})$$

$$\therefore AC = \frac{AM \times AB}{BM}$$

$$\therefore AC = \frac{8 \times 8\sqrt{3}}{16} = 4\sqrt{3} \text{ cm.} \quad (\text{Second req.})$$

4

[a] $\therefore \overline{AB}, \overline{AC}$ are two tangent-segments to the circle

$$\therefore AB = AC$$

$$\therefore \text{In } \triangle ABC: m(\angle ABC) = m(\angle ACB) = \frac{180^\circ - 50^\circ}{2} = 65^\circ$$

$\therefore BCDE$ is a cyclic quadrilateral.

$$\therefore m(\angle EBC) + m(\angle D) = 180^\circ$$

$$\therefore m(\angle EBC) = 180^\circ - 115^\circ = 65^\circ$$

$$\therefore m(\angle ABC) = m(\angle EBC)$$

$$\therefore \overline{BC} \text{ bisects } \angle ABE \quad (\text{Q.E.D.1})$$

$$\therefore m(\angle BEC) \text{ (inscribed)} = m(\angle ABC) \text{ (tangency)} = 65^\circ$$

$$\therefore m(\angle EBC) = m(\angle BEC)$$

$$\therefore \text{In } \triangle BCE: CB = CE \quad (\text{Q.E.D.2})$$

[b] $\therefore \angle ABE$ is an exterior angle of the cyclic quadrilateral $ABCD$

$$\therefore m(\angle D) = m(\angle ABE) = 100^\circ$$

In $\triangle ACD$:

$$\therefore m(\angle ACD) = 180^\circ - (100^\circ + 40^\circ) = 40^\circ$$

$$\therefore m(\angle ACD) = m(\angle CAD)$$

$$\therefore CD = AD$$

$$\therefore m(\widehat{CD}) = m(\widehat{AD}) \quad (\text{Q.E.D.})$$

5

$$[a] \therefore m(\angle ACB) = \frac{1}{2} m(\angle AMB) = 60^\circ$$

(inscribed and central angles subtended the same arc \widehat{AB})

(1)

$$\therefore \overline{CD} \parallel \overline{AB}$$

$$\therefore m(\widehat{AC}) = m(\widehat{BC})$$

$$\therefore AC = BC$$

(2)

From (1) and (2):

$$\therefore \triangle CAB \text{ is an equilateral triangle.} \quad (\text{Q.E.D.})$$

[b] In $\triangle ADE, ACE$

$$\begin{cases} m(\angle DAE) = m(\angle CAE) \\ AD = AC \\ \overline{AE} \text{ is a common side} \end{cases}$$

$$\therefore \triangle ADE \cong \triangle ACE$$

$$\therefore m(\angle ADE) = m(\angle ACE)$$

$$\therefore m(\angle ADE) = m(\angle ACE)$$

$$\therefore m(\angle AFB) = m(\angle ACB)$$

(two inscribed angles subtended by \widehat{AB})

$$\therefore m(\angle ADE) = m(\angle EFB)$$

$$\therefore DBFE \text{ is a cyclic quadrilateral.} \quad (\text{Q.E.D.})$$

3 - Alexandria

1

1 b

2 d

3 a

4 b

5 d

6 c

2

[a] $\therefore \overline{CD}$ is a diameter in a circle M

$$\therefore AB = 10 \text{ cm, } \therefore \overline{MH} \perp \overline{AB}$$

$$\therefore AH = BH = 5 \text{ cm.}$$

$$\text{In } \triangle AHM: \therefore m(\angle AMH) = 30^\circ$$

$$\therefore m(\angle AHM) = 90^\circ$$

$$\therefore AM = 2AH = 10 \text{ cm}$$

$$\therefore CD = 2 \times 10 = 20 \text{ cm.} \quad (\text{The req.})$$

Geometry

[b] ∴ The figure

ABCD is a cyclic quadrilateral.

$$\therefore m(\angle ABC) = 180^\circ - 125^\circ = 55^\circ$$

∴ \overline{EA} , \overline{EB} are two tangents to the circle at A and B

$$\therefore EA = EB$$

$$\therefore m(\angle EAB) = \frac{180^\circ - 70^\circ}{2} = 55^\circ$$

∴ \overline{EA} is a tangent to the circle at A

$$\therefore m(\angle EAB) \text{ (tangency)}$$

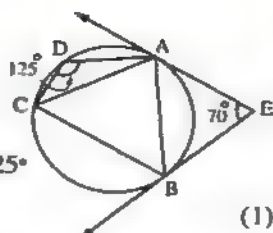
$$= m(\angle ACB) \text{ (inscribed)} = 55^\circ$$

From (1) and (2):

$$\therefore m(\angle ACB) = m(\angle ABC) = 55^\circ$$

$$\therefore AB = AC$$

(Q.E.D.)



3

$$[a] \therefore m(\angle A) = \frac{1}{2} [m(\widehat{HC}) - m(\widehat{BD})]$$

$$\therefore 30^\circ = \frac{1}{2} [120^\circ - m(\widehat{BD})]$$

$$\therefore 60^\circ = 120^\circ - m(\widehat{BD})$$

$$\therefore m(\widehat{BD}) = 120^\circ - 60^\circ = 60^\circ$$

(First req.)

$$\therefore m(\widehat{BC}) = m(\widehat{DH})$$

$$\therefore BC = DH$$

By adding $m(\widehat{BD})$ to both sides.

$$\therefore m(\widehat{CD}) = m(\widehat{HB}) \quad \therefore m(\angle C) = m(\angle H)$$

In $\triangle ACH$: $\therefore AC = AH$

$$\therefore BC = DH$$

By subtracting: $AB = AD$

(Second req.)

[b] In $\triangle ABD$:

$$\therefore AB = AD$$

$$\therefore m(\angle ABD) = m(\angle ADB) = 30^\circ$$

$$\therefore m(\angle A) = 180^\circ - 2 \times 30^\circ = 120^\circ$$

$$\therefore m(\angle A) + m(\angle C) = 120^\circ + 60^\circ = 180^\circ$$

∴ ABCD is a cyclic quadrilateral.

(Q.E.D.)

4

$$[a] \therefore m(\angle ACB) = \frac{1}{2} m(\angle AMB) = 60^\circ$$

(inscribed and central angles subtended the same arc \widehat{AB})

(1)

$$\therefore \overline{CD} \parallel \overline{AB}$$

$$\therefore m(\widehat{AC}) = m(\widehat{BC})$$

$$\therefore AC = BC$$

(2)

From (1) and (2):

∴ $\triangle CAB$ is equilateral.

(Q.E.D.)

[b] Construction:

Draw \overline{AB}

Proof:

∴ The figure ABCD is a cyclic quadrilateral

$$\therefore m(\angle BAD) = 180^\circ - 70^\circ = 110^\circ$$

∴ The figure ABFE is a cyclic quadrilateral and $\angle BAD$ is an exterior angle of it

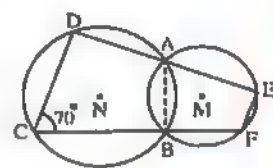
$$\therefore m(\angle F) = m(\angle BAD) = 110^\circ$$

$$\therefore m(\angle F) + m(\angle C) = 110^\circ + 70^\circ = 180^\circ$$

but they are two interior angles on the same side of the transversal \overline{FC}

$$\therefore \overline{CD} \parallel \overline{EF}$$

(Q.E.D.)



5

[a] In $\triangle ABC$:

$$\therefore AC = BC$$

$$\therefore m(\angle BAC) = m(\angle ABC) = 65^\circ$$

$$\therefore m(\angle CAD) = 130^\circ - 65^\circ = 65^\circ$$

$$\therefore m(\angle B) = m(\angle CAD) = 65^\circ$$

∴ \overline{AD} is a tangent to the circle passing through the vertices of the triangle ABC

(Q.E.D.)

[b] ∴ \overline{AD} is a tangent

$$\therefore \overline{MD} \perp \overline{AD}$$

$$\therefore m(\angle MDA) = 90^\circ$$

∴ H is the midpoint of \overline{BC}

$$\therefore \overline{MH} \perp \overline{BC}$$

$$\therefore m(\angle MHA) = 90^\circ$$

From the quadrilateral ADMH:

$$\therefore m(\angle DMH) = 360^\circ - (56^\circ + 90^\circ + 90^\circ) = 124^\circ$$

(The req.)

Answers of Final Examinations

4 El-Kalyoubia

1

- 1 c 2 a 3 d 4 b 5 d 8 c

2

[a] $\because \overline{AB} \parallel \overline{CD} \therefore m(\widehat{AC}) = m(\widehat{BD}) = 50^\circ$

$$\therefore m(\angle BED) = \frac{1}{2} m(\widehat{BD})$$

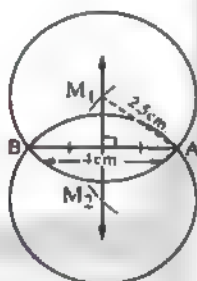
$$\therefore (3y - 5)^\circ = \frac{1}{2} \times 50^\circ = 25^\circ$$

$$\therefore 3y = 5^\circ + 25^\circ = 30^\circ$$

$$\therefore y = 10^\circ$$

(The req.)

[b]

 \therefore We can draw two circles.

3

[a] $\because X$ is a midpoint of \overline{AB}

$$\therefore \overline{MX} \perp \overline{AB}$$

$$\therefore m(\angle MXA) = 90^\circ (1)$$

 $\because Y$ is a midpoint of \overline{AC}

$$\therefore \overline{MY} \perp \overline{AC}$$

$$\therefore m(\angle MYA) = 90^\circ (2)$$

From (1) and (2):

$$\therefore m(\angle MXA) = m(\angle MYA)$$

but they are drawn on \overline{AM} and on one side of it.

$$\therefore AXYM \text{ is a cyclic quadrilateral. (Q.E.D.1)}$$

In $\triangle MAC$: $\therefore MA = MC = r$

$$\therefore m(\angle MCA) = m(\angle MAC)$$

 $\therefore AXYM$ is a cyclic quadrilateral.

$$\therefore m(\angle MXY) = m(\angle MAY)$$

$$\therefore m(\angle MXY) = m(\angle MCY) \quad (\text{Q.E.D.2})$$

[b] $\because ABCD$ is a cyclic quadrilateral

$$\therefore m(\angle A) + m(\angle C) = 180^\circ$$

$$\therefore m(\angle C) = 180^\circ - 120^\circ = 60^\circ \quad (\text{First req.})$$

$$\therefore m(\angle FBC) = m(\angle C) = 60^\circ \text{ (alternate angles)}$$

$$\therefore m(\angle EBC) = 65^\circ + 60^\circ = 125^\circ$$

 $\therefore ABCD$ is a cyclic quadrilateral.

$$\therefore m(\angle D) = m(\angle EBC) = 125^\circ \quad (\text{Second req.})$$

4

[a] \because The circle $M \cap$ The circle $N = \{A, B\}$ $\therefore \overline{MN}$ is the axis of symmetry of \overline{AB} \therefore In $\triangle ABD$: \overline{DC} is the axis of symmetry of \overline{AB}

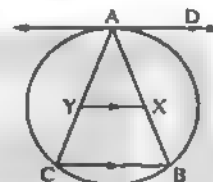
$$\therefore AD = BD$$

$$\therefore \overline{MX} \perp \overline{AD}, \overline{MY} \perp \overline{BD}$$

$$\therefore MX = MY$$

(Q.E.D.)

[b]

 $\therefore \overline{AD}$ is a tangent to the circle.

$$\therefore m(\angle DAB) \text{ (tangency)} = m(\angle ACB)$$

(inscribed) (1)

 $\because \overline{XY} \parallel \overline{BC}$, \overline{YC} is a transversal

$$\therefore m(\angle AYX) = m(\angle ACB)$$

(corresponding angles) (2)

$$\therefore \text{From (1) and (2): } \therefore m(\angle DAB) = m(\angle AYX)$$

 $\therefore \overline{AD}$ is a tangent to the circle passing

through the points A, X and Y (Q.E.D.)

5

[a] $\because \overline{AC}$ and \overline{AB} are two tangent-segments to the circle M

$$\therefore \overline{AE} \perp \overline{BC}$$

$$\therefore m(\angle CEM) = 90^\circ$$

 $\because \overline{BD}$ is a diameter in the circle M

$$\therefore m(\angle ECD) = 90^\circ$$

$$\therefore m(\angle CEM) + m(\angle ECD) = 180^\circ$$

 \therefore but they are two interior angles in the same side of the transversal \overline{BC}

$$\therefore \overline{AM} \parallel \overline{CD}$$

(Q.E.D.)

131

Geometry

[b] $\because \overline{CM} \parallel \overline{AB}$, \overline{MA} is a transversal.

$$\therefore m(\angle MAB) = m(\angle AMC) \text{ (alternate angles)}$$

$$\therefore m(\angle AMC) = 2m(\angle B)$$

(central and inscribed angles subtended by \widehat{AC})

$$\therefore m(\angle EAB) = 2m(\angle B)$$

$$\therefore m(\angle EAB) > m(\angle B)$$

From $\triangle EAB$: $\therefore BE > AE$ (Q.E.D.)

5 El-Sharkia

1

1 b 2 a 3 d 4 b 5 d 6 a

2

[a] $\because X$ is the midpoint of \overline{AB}

$$\therefore \overline{MX} \perp \overline{AB}$$

$\because Y$ is the midpoint of \overline{AC}

$$\therefore \overline{MY} \perp \overline{AC}$$

$$\because AB = AC \quad \therefore MX = MY$$

$$\because MD = ME = r \quad \therefore XD = YE \text{ (Q.E.D.)}$$

[b] $\because \overline{AB}$, \overline{AC} are two tangent-segments to the circle.

$$\therefore AB = AC$$

\therefore In $\triangle ABC$:

$$m(\angle ABC) = m(\angle ACB) = \frac{180^\circ - 70^\circ}{2} = 55^\circ$$

$\because BCDE$ is a cyclic quadrilateral.

$$\therefore m(\angle EBC) + m(\angle D) = 180^\circ$$

$$\therefore m(\angle EBC) = 180^\circ - 125^\circ = 55^\circ$$

$$\therefore m(\angle ABC) = m(\angle EBC)$$

$\therefore \overline{BC}$ bisects $\angle ABE$ (Q.E.D.)

3

[a] $\because ABDC$ is a cyclic quadrilateral.

$$\therefore m(\angle A) + m(\angle D) = 180^\circ$$

$$\therefore m(\angle A) = 180^\circ - 140^\circ = 40^\circ$$

$\because \overline{AB}$ is a diameter.

$$\therefore m(\angle ACB) = 90^\circ$$

In $\triangle ABC$:

$$\therefore m(\angle ABC) = 180^\circ - (90^\circ + 40^\circ) = 50^\circ \text{ (First req.)}$$

$$\because m(\widehat{BD}) = m(\widehat{DC}) \quad \therefore BD = CD$$

In $\triangle BCD$:

$$\therefore m(\angle CBD) = m(\angle BCD) = \frac{180^\circ - 140^\circ}{2} = 20^\circ$$

$$\therefore m(\widehat{BD}) = 2m(\angle BCD) = 2 \times 20^\circ = 40^\circ$$

$$\therefore m(\widehat{AB}) = 180^\circ$$

$$\therefore m(\widehat{ABD}) = 180^\circ + 40^\circ = 220^\circ \text{ (Second req.)}$$

[b] $\because \overline{AD}$ is a tangent to the circle

$$\therefore m(\angle DAB) \text{ (tangen } y) = m(\angle ACB) \text{ (inscribed)} \quad (1)$$

$\because \overline{XY} \parallel \overline{BC}$, \overline{YC} is a transversal.

$$\therefore m(\angle AYX) = m(\angle ACB) \text{ (corresponding angles)} \quad (2)$$

\therefore From (1) and (2):

$$\therefore m(\angle DAB) = m(\angle AYX)$$

$\therefore \overline{AD}$ is a tangent to the circle which passes through the points A, X and Y (Q.E.D.)

4

$$[a] \because m(\widehat{BD}) = 2m(\angle DCB) = 2 \times 25^\circ = 50^\circ$$

$\because D$ is midpoint of \widehat{AB}

$$\therefore m(\widehat{AB}) = 2 \times 50^\circ = 100^\circ$$

$$\therefore m(\angle AMB) = m(\widehat{AB}) = 100^\circ \text{ (The req.)}$$

[b] $\because \triangle ABC$ is equilateral.

$$\therefore m(\angle B) = 60^\circ$$

$$\therefore m(\angle D) = m(\angle B) = 60^\circ$$

(two inscribed angles subtended by \widehat{AC})

$$\therefore AD = DE$$

$\therefore \triangle ADE$ is an equilateral triangle. (Q.E.D.1)

$$\therefore m(\angle DAE) = m(\angle BAC) = 60^\circ$$

Subtracting $\angle BAE$ from both sides.

$$\therefore m(\angle DAB) = m(\angle EAC) \text{ (Q.E.D.2)}$$

5

[a] $\because \overline{AB}$ is a tangen-tsegment to the circle.

$$\therefore \overline{MA} \perp \overline{AB} \quad \therefore m(\angle A) = 90^\circ$$

$$\text{In } \triangle MAB: \tan(\angle B) = \frac{AM}{AB}$$

$$\therefore \tan 30^\circ = \frac{8}{AB}$$

$$\therefore AB = \frac{8}{\tan 30^\circ} = 8\sqrt{3} \text{ cm.}$$

Answers of Final Examinations

In $\triangle MAB$: $\therefore m(\angle AMB) = 180^\circ - (90^\circ + 30^\circ)$
 $= 60^\circ$

$\therefore m(\angle XAB) = \frac{1}{2} m(\angle AMB)$

(tangency and central angles)

$\therefore m(\angle XAB) = \frac{1}{2} \times 60^\circ = 30^\circ$

In $\triangle XAB$:

$\therefore m(\angle XAB) = m(\angle XBA)$

$\therefore \triangle XAB$ is an isosceles triangle. (Second req.)

[b] In $\triangle ADE$, $\triangle ACE$:

$\begin{cases} m(\angle DAE) = m(\angle CAE) \\ AD = AC \\ \overline{AE} \text{ is a common side} \end{cases}$

$\therefore \triangle ADE \cong \triangle ACE$

$\therefore m(\angle ADE) = m(\angle ACE)$

$\therefore m(\angle AFB) = m(\angle ACB)$

(two inscribed angles subtended by \widehat{AB})

$\therefore m(\angle ADE) = m(\angle EFB)$

$\therefore DBFE$ is a cyclic quadrilateral. (Q.E.D.)

El-Monofia

1

1 c 2 a 3 b 4 b 5 c 6 b

2

[a] $\therefore \overline{AB}$, \overline{AC} are two tangent-segments to the circle M

$\therefore \overline{AB} \perp \overline{MB}$, $\overline{AC} \perp \overline{MC}$

$\therefore m(\angle BAC) = 360^\circ - (90^\circ + 90^\circ + 90^\circ) = 90^\circ$

$\therefore MB = MC = r$

$\therefore ABMC$ is a square. (Q.E.D.)

[b] In $\triangle AMB$: $\therefore AM = MB = r$

$\therefore m(\angle MAB) = m(\angle ABM)$

$\therefore m(\angle CAB) = m(\angle MAB)$

$\therefore m(\angle CAB) = m(\angle ABM)$ and they are alternate angles.

$\therefore \overline{AC} \parallel \overline{BM}$

$\therefore D$ is the midpoint of \overline{AC}

$\therefore \overline{MD} \perp \overline{AC}$

$\therefore \overline{AC} \parallel \overline{BM}$

$\therefore \overline{DM} \perp \overline{BM}$

(Q.E.D.)

3

[a] $\therefore \overline{AX}$, \overline{AZ} are two tangent-segments

$\therefore AX = AZ = 6 \text{ cm.} \quad \therefore AC = 10 \text{ cm.}$

$\therefore CZ = 10 - 6 = 4 \text{ cm.}$

$\therefore \overline{CY}$, \overline{CZ} are two tangent-segments

$\therefore CY = CZ = 4 \text{ cm.}$

$\therefore \overline{BX}$, \overline{BY} are two tangent-segments

$\therefore BX = BY$

\therefore The perimeter of $\triangle ABC = 24 \text{ cm.}$

$\therefore BX + BY + 6 + 10 + 4 = 24$

$\therefore BX + BY = 4 \quad \therefore BX = 2 \text{ cm.}$

$\therefore AB = 6 + 2 = 8 \text{ cm.} \quad \text{(First req.)}$

$\therefore (AC)^2 = (10)^2 = 100$

$\therefore (AB)^2 + (BC)^2 = (8)^2 + (6)^2 = 100 = (AC)^2$

$\therefore \triangle ABC$ is a right-angled triangle at B
 (Second req.)

[b] $\therefore m(\widehat{AX}) = m(\widehat{AY})$

$\therefore m(\angle ACX) = m(\angle ABY)$

and they are drawn on

\overline{DE} and on one side of it

\therefore The figure $BCED$ is a cyclic quadrilateral.

(Q.E.D.1)

$\therefore m(\angle DEB) = m(\angle DCB)$

(drawn on \overline{DB} and on one side of it)

$\therefore m(\angle XAB) = m(\angle XCB)$

(two inscribed angles subtended by \widehat{XB})

$\therefore m(\angle DEB) = m(\angle XAB) \quad \text{(Q.E.D.2)}$

4

[a] In $\triangle ABC$: $\therefore CA = CB$

(1)

$\therefore m(\angle A) = m(\angle B) \quad \therefore \sin A = \sin B$

$\therefore \frac{XM}{AM} = \frac{YM}{BM} \quad \therefore AM = BM = r$

$\therefore XM = YM$

$\therefore \overline{MX} \perp \overline{DA}$, $\overline{MY} \perp \overline{EB}$

$\therefore DA = EB$

(2)

Subtracting (2) from (1) : $\therefore CD = CE \quad \text{(Q.E.D.)}$

[b] $\therefore \overline{AB}$ is a diameter in the circle M

$\therefore m(\angle ACB) = 90^\circ$

$\therefore \overline{ED} \perp \overline{AB}$

$\therefore m(\angle FDA) = 90^\circ$

Geometry

$$\therefore m(\angle ACF) + m(\angle FDA) = 90^\circ + 90^\circ = 180^\circ$$

\therefore The figure ADFC is a cyclic quadrilateral.

(Q.E.D.1)

$\therefore \overline{EC}$ is a tangent of the circle M

$$\therefore m(\angle ECB) \text{ (tangency)} = m(\angle CAB) \text{ (inscribed)}$$

$\therefore \angle CFE$ is an exterior angle of the cyclic quadrilateral ADFC

$$\therefore m(\angle CAB) = m(\angle CFE)$$

$$\therefore m(\angle ECF) = m(\angle CFE)$$

In $\triangle ECF$: $\therefore \triangle ECF$ is an isosceles triangle.

(Q.E.D.2)

5

[a] Construction :

Draw \overline{MD}

Proof :

$\therefore \overline{BM}$ is a diameter in the circle N

$$\therefore m(\angle MDB) = 90^\circ \quad \therefore \overline{MD} \perp \overline{BC}$$

$$\therefore CD = DB = 4 \text{ cm.} \quad \therefore MB = AM = 5 \text{ cm.}$$

In $\triangle ABC$:

$$\begin{aligned} \therefore (AC)^2 &= (AB)^2 - (BC)^2 = (10)^2 - (8)^2 \\ &= 100 - 64 = 36 \end{aligned}$$

$$\therefore AC = 6 \text{ cm.} \quad \text{(The req.)}$$

[b] $\therefore \overline{AD}$ is a tangent to the circle

$$\begin{aligned} \therefore m(\angle DAB) \text{ (tangency)} \\ = m(\angle ACB) \text{ (inscribed)} \end{aligned} \quad (1)$$

$\therefore \overline{XY} \parallel \overline{BC}$, \overline{YC} is a transversal

$$\begin{aligned} \therefore m(\angle AYX) &= m(\angle ACB) \\ \text{(corresponding angles)} \end{aligned} \quad (2)$$

From (1) and (2):

$$\therefore m(\angle DAB) = m(\angle AYX)$$

$\therefore \overline{AD}$ is a tangent to the circle passing through the points A, X and Y (Q.E.D.)

7 El-Gharbia

1

- 1 b 2 a 3 d 4 c 5 b 6 d

2

[a] $\therefore \overline{AB} \parallel \overline{CD}$, \overline{AD} is a transversal

$$\begin{aligned} \therefore m(\angle ADC) &= m(\angle BAD) = 20^\circ \\ \text{(alternate angles)} \end{aligned}$$

$$\begin{aligned} \therefore m(\angle AEC) &= m(\angle ADC) = 20^\circ \\ \text{(two inscribed angles subtended by } \widehat{AC}) \end{aligned}$$

$$\therefore 3x - 7 = 20 \quad \therefore 3x = 27$$

$$\therefore x = 9 \quad \text{(The req.)}$$

[b] $\therefore \overline{BD}$ is a tangent-segment to the circle

$$\therefore m(\angle ABD) = 90^\circ$$

$\therefore E$ is the midpoint of \overline{AC}

$$\therefore \overline{ME} \perp \overline{AC} \quad \therefore m(\angle MED) = 90^\circ$$

$$\therefore m(\angle MBD) + m(\angle MED) = 90^\circ + 90^\circ = 180^\circ$$

\therefore The figure MEDB is a cyclic quadrilateral. (Q.E.D.1)

$\therefore \angle BMX$ is an exterior angle of the cyclic quadrilateral MEDB

$$\therefore m(\angle D) = m(\angle BMX)$$

$$\therefore m(\angle BAX) = \frac{1}{2} m(\angle BMX)$$

(inscribed and central angles subtended by \widehat{XB})

$$\therefore m(\angle BAX) = \frac{1}{2} m(\angle D) \quad \text{(Q.E.D.2)}$$

3

[a] In $\triangle ABC$: $\therefore m(\angle BAC) = 90^\circ$

$$\therefore \tan B = \frac{5}{5\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\therefore m(\angle B) = 30^\circ$$

$$\therefore m(\angle ABC) = m(\angle DAC) = 30^\circ$$

$\therefore \overline{AD}$ is a tangent to the circle passing through the vertices of $\triangle ABC$ (Q.E.D.)

[b] $\therefore \overline{AB}$, \overline{AC} are two tangent-segments of the circle

$$\therefore AB = AC$$

$$\therefore m(\angle ABC) = m(\angle ACB) \quad (1)$$

$\therefore \overline{AB} \parallel \overline{CD}$ and \overline{BC} is a transversal

$$\begin{aligned} \therefore m(\angle BCD) &= m(\angle ABC) \\ \text{(alternate angles)} \end{aligned} \quad (2)$$

From (1) and (2): $\therefore m(\angle BCD) = m(\angle ACB)$

$\therefore \overline{CB}$ bisects $\angle ACD$ (Q.E.D.)

4

[a] $\because \angle AMB$ is an exterior angle of the $\triangle AMD$

$$\therefore m(\angle AMB) = m(\angle ADM) + m(\angle DAM)$$

$$\therefore 80^\circ = 30^\circ + m(\angle DAM)$$

$$\therefore m(\angle DAM) = 80^\circ - 30^\circ = 50^\circ$$

In $\triangle ADC$: $\because DA = DC$

$$\therefore m(\angle DCA) = m(\angle DAC) = 50^\circ$$

$$\therefore m(\angle ABD) = m(\angle ACD)$$

and they are drawn on \overline{AD} and on one side of it \therefore The figure ABCD is a cyclic quadrilateral.

(Q.E.D.)

[b] $\because X$ is the midpoint of \overline{AB}

$$\therefore \overline{MX} \perp \overline{AB}$$

 $\because Y$ is the midpoint of \overline{AC}

$$\therefore \overline{MY} \perp \overline{AC}$$

$$\because AB = AC \quad \therefore MX = MY$$

$$\because MD = ME = r$$

$$\therefore XD = YE$$

(Q.E.D.)

5

[a] $\because m(\widehat{AD}) = 2 m(\angle ABD) = 2 \times 22^\circ = 44^\circ$

$$\because m(\angle C) = \frac{1}{2} [m(\widehat{BE}) - m(\widehat{AD})]$$

$$\therefore 36^\circ = \frac{1}{2} [m(\widehat{BE}) - 44^\circ]$$

$$\therefore 72^\circ = m(\widehat{BE}) - 44^\circ$$

$$\therefore m(\widehat{BE}) = 116^\circ$$

(The req.)

[b] $\because m(\angle BDC) = m(\angle BAC)$ (two inscribed angles subtended by \widehat{BC})

$$\therefore m(\angle BDC) = 30^\circ$$

(First req.)

$$\therefore m(\widehat{BC}) = 2 m(\angle BAC) = 60^\circ$$

 $\because \overline{AB}$ is diameter in the circle M

$$\therefore m(\widehat{AB}) = 180^\circ$$

$$\therefore m(\widehat{AC}) = 180^\circ - 60^\circ = 120^\circ$$

 $\because D$ is the midpoint of \widehat{AC}

$$\therefore m(\widehat{AD}) = \frac{120^\circ}{2} = 60^\circ$$

$$\therefore m(\angle ACD) = \frac{1}{2} m(\widehat{AD}) = \frac{1}{2} \times 60^\circ = 30^\circ$$

 $\therefore m(\angle BAC) = m(\angle ACD)$ but they are alternate angles

$$\therefore \overline{DC} \parallel \overline{AB}$$

(Second req.)

8

El-Dakahlia

1

[a] 1 a

2 d

3 c

[b] $\because \angle ABH$ is an exterior angle of the cyclic quadrilateral ABCD

$$\therefore m(\angle ADC) = m(\angle ABH) = 110^\circ$$

In $\triangle ACD$:

$$\therefore m(\angle ACD) = 180^\circ - (110^\circ + 35^\circ) = 35^\circ$$

$$\therefore m(\angle CAD) = m(\angle ACD) \quad \therefore CD = AD$$

$$\therefore m(\widehat{CD}) = m(\widehat{AD})$$

(Q.E.D.)

2

[a] 1 c

2 a

3 d

[b] $\because \overline{AB}, \overline{AC}$ are two tangents to the circle

$$\therefore AB = AC$$

$$\therefore m(\angle ABC) = m(\angle ACB) = \frac{180^\circ - 70^\circ}{2} = 55^\circ$$

(First req.)

 $\because BCHD$ is a cyclic quadrilateral

$$\therefore m(\angle C) + m(\angle D) = 180^\circ$$

$$\therefore m(\angle C) = 180^\circ - 125^\circ = 55^\circ$$

$$\therefore m(\angle BHC) \text{ (inscribed)}$$

$$= m(\angle ABC) \text{ (tangency)} = 55^\circ$$

$$\therefore m(\angle BCH) = m(\angle BHC)$$

In $\triangle BCH$: $\therefore CB = BH$

(Second req.)

3

[a] Construction :

Draw \overline{MC}

Proof :

$$\because \overline{CD} \parallel \overline{AB}, \overline{MY} \text{ is a transversal}$$

$$\therefore m(\angle MXC) + m(\angle XMA) = 180^\circ$$

$$\therefore m(\angle MXC) = 90^\circ$$

$$\because MX = \frac{1}{2} MY, MY = MC$$

$$\therefore MX = \frac{1}{2} MC \quad \therefore m(\angle MCX) = 30^\circ$$

$$\therefore m(\angle AMC) = m(\angle MCX) = 30^\circ$$

(alternate angles)

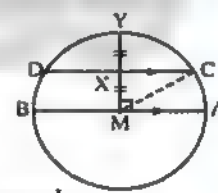
$$\therefore m(\widehat{AC}) = m(\angle AMC) = 30^\circ$$

(First req.)

$$\because \widehat{AY} = m(\angle AMY) = 90^\circ$$

$$\therefore m(\widehat{CY}) = 90^\circ - 30^\circ = 60^\circ$$

(Second req.)



Geometry

- [b] $\because AB = AC$
 $\therefore \overline{MD} \perp \overline{AB}, \overline{MH} \perp \overline{AC}$
 $\therefore MD = MH$
 $\therefore MX = MY = r \quad \therefore XD = HY \text{ (Q.E.D.)}$

4

- [a] $\because \overline{AO} \parallel \overline{DH}, \overline{AH}$ is a transversal
 $\therefore m(\angle HAO) = m(\angle AHD)$ (alternate angles) (1)
 $\therefore m(\angle C)$ (inscribed)
 $= m(\angle BAO)$ (tangency) (2)
 From (1) and (2):
 $\therefore m(\angle C) = m(\angle AHD)$
 $\therefore DHBC$ is a cyclic quadrilateral (Q.E.D.)

[b] Construction :

Draw $\overline{MA}, \overline{MC}$

Proof :

- $\because \overline{AB}$ touches the smaller circle at C
 $\therefore \overline{MC} \perp \overline{AB}$
 $\therefore \overline{AB}$ is a chord of the greater circle
 $\therefore \overline{MC} \perp \overline{AB}$
 $\therefore C$ is the midpoint of \overline{AB}
 $\therefore AC = \frac{14}{2} = 7 \text{ cm.}$
 $\therefore \triangle AMC$ is a right-angled at C
 $\therefore (AC)^2 = (MA)^2 - (MC)^2$
 $\therefore (7)^2 = r_1^2 - r_2^2 \quad \therefore r_1^2 - r_2^2 = 49$
 \therefore The area of the part included between the two circles = The area of the greater circle - The area of the smaller circle = $\pi r_1^2 - \pi r_2^2 = \pi (r_1^2 - r_2^2)$
 $= \frac{22}{7} \times 49 = 154 \text{ cm}^2$ (The req.)

5

- [a] $\because m(\angle ACB) = \frac{1}{2} m(\angle AMB)$
 (inscribed and central angles subtended the same arc \widehat{AB})
 $\therefore m(\angle ACB) = \frac{1}{2} \times 120^\circ = 60^\circ$ (1)
 $\because \overline{CD} \parallel \overline{AB} \quad \therefore m(\widehat{AC}) = m(\widehat{BC})$
 $\therefore AC = BC$ (2)
 From (1) and (2):
 $\therefore \triangle ABC$ is equilateral (Q.E.D.)

[b] Construction :

Draw \overline{MB}

Proof :

In $\triangle MAB$:

$$\because MA = MB = r, m(\angle MAB) = 60^\circ$$

 $\therefore \triangle AMB$ is equilateral

$$\therefore m(\angle AMB) = 60^\circ \quad (1)$$

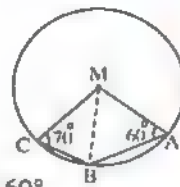
In $\triangle MBC$: $\because MB = MC = r$

$$\therefore m(\angle MBC) = m(\angle MCB) = 70^\circ$$

$$\therefore m(\angle CMB) = 180^\circ - (70^\circ + 70^\circ) = 40^\circ \quad (2)$$

From (1) and (2) :

$$\therefore m(\angle AMC) = m(\angle AMB) + m(\angle CMB) = 60^\circ + 40^\circ = 100^\circ \quad (\text{The req.})$$



9

Ismailia

1

- 1 c 2 b 3 c 4 a 5 d 6 b

2

- [a] $\because m(\angle A) = \frac{1}{2} m(\angle BMC) = x^\circ$
 (inscribed and central angles subtended by \widehat{BC})
 \therefore The figure $ABDC$ is a cyclic quadrilateral
 $\therefore m(\angle A) + m(\angle BDC) = 180^\circ$
 $\therefore x + 2x = 180^\circ \quad \therefore 3x = 180^\circ$
 $\therefore x = 60^\circ \quad \therefore m(\angle A) = 60^\circ$ (The req.)
 [b] $\because m(\angle A) = m(\angle B)$
 (two inscribed angles subtended by \widehat{CD})
 $\therefore m(\angle C) = m(\angle D)$
 (two inscribed angles subtended by \widehat{AB})
 $\therefore EA = ED \quad \therefore m(\angle A) = m(\angle D)$
 $\therefore m(\angle C) = m(\angle B)$
 $\therefore EB = EC$ (Q.E.D.)

3

[a] In $\triangle ABC$:

$$\therefore AB = AC$$

$$\therefore m(\angle ABC) = m(\angle ACB)$$

 $\therefore \overline{BX}$ bisects $(\angle ABC)$, \overline{CY} bisects $(\angle ACB)$

$$\therefore m(\angle XBY) = m(\angle YCX)$$

and they are drawn on \overline{XY} and on one side of it $\therefore BCXY$ is a cyclic quadrilateral (Q.E.D.)

Geometry

[b] $\therefore ABC$ is an equilateral triangle

$$\therefore m(\angle A) = 60^\circ$$

$\therefore m(\angle D) = m(\angle A)$ and they are drawn on \overline{BC} and on one side of it

$\therefore ABCD$ is a cyclic quadrilateral. (Q.E.D.)

5

[a] $m(\widehat{AB}) = 2 m(\angle ADB) = 60^\circ$ (First req.)

$$m(\angle DCB) = \frac{1}{2} [m(\widehat{AD}) + m(\widehat{AB})] \\ = \frac{1}{2} [90^\circ + 60^\circ] = 75^\circ \text{ (Second req.)}$$

[b] $\therefore \overline{AB}, \overline{AC}$ are two tangents to the circle.

$$\therefore AB = AC$$

\therefore In $\triangle ABC$:

$$m(\angle ABC) = m(\angle ACB) = \frac{1}{2} (180^\circ - 40^\circ) = 70^\circ \text{ (First req.)}$$

$\therefore \overline{AB} \parallel \overline{CD}, \overline{BC}$ is a transversal

$$\therefore m(\angle BCD) = m(\angle ABC) = 70^\circ \quad (1)$$

(alternate angles)

$$\therefore m(\angle BDC) \text{ (inscribed)} \\ = m(\angle ABC) \text{ (tangency)} = 70^\circ \quad (2)$$

From (1) and (2):

$$\therefore m(\angle BCD) = m(\angle BDC)$$

\therefore In $\triangle BCD$: $BC = BD$ (Second req.)

11 Port Said

1

- 1 d 2 c 3 b 4 b 5 a 6 b

2

[a] $\therefore MF = ME$ (lengths of two radii)

$$\therefore XF = YE \quad \therefore MX = MY$$

$$\therefore \overline{MX} \perp \overline{AB}, \overline{MY} \perp \overline{CD}$$

$$\therefore AB = CD \quad (Q.E.D.1)$$

$$\therefore \overline{MX} \perp \overline{AB}$$

$\therefore X$ is the midpoint of \overline{AB}

$$\therefore AX = \frac{1}{2} AB \quad \therefore \overline{MY} \perp \overline{CD}$$

$\therefore Y$ is the midpoint of \overline{CD}

$$\therefore CY = \frac{1}{2} CD \quad \therefore AB = CD$$

$$\therefore AX = CY$$

\therefore In $\triangle AXF, CYE$

$$\begin{cases} AX = CY \\ XF = YE \end{cases}$$

$$\therefore m(\angle AXF) = m(\angle CYE) = 90^\circ$$

$$\therefore \triangle AXF \cong \triangle CYE, AF = CE \quad (Q.E.D.2)$$

[b] $\therefore m(\angle A) = \frac{1}{2} [m(\widehat{CE}) - m(\widehat{BD})]$

$$\therefore 30^\circ = \frac{1}{2} [120^\circ - m(\widehat{BD})]$$

$$\therefore 60^\circ = 120^\circ - m(\widehat{BD})$$

$$\therefore m(\widehat{BD}) = 120^\circ - 60^\circ = 60^\circ \quad (\text{The req.})$$

3

[a] In $\triangle ABC$: $\therefore m(\angle BAC) = 90^\circ, AC = \frac{1}{2} BC$

$$\therefore m(\angle B) = 30^\circ$$

$$\therefore m(\angle C) = 180^\circ - (90^\circ + 30^\circ) = 60^\circ$$

$$\therefore m(\angle C) = m(\angle DAB) = 60^\circ$$

$\therefore \overline{AD}$ is a tangent to the circle passing through the vertices of $\triangle ABC$ (Q.E.D.)

[b] $\therefore D$ is the midpoint of \overline{AB}

$$\therefore \overline{MD} \perp \overline{AB} \quad \therefore m(\angle ADM) = 90^\circ$$

$\therefore E$ is the midpoint of \overline{AC}

$$\therefore \overline{ME} \perp \overline{AC} \quad \therefore m(\angle AEM) = 90^\circ$$

From the quadrilateral $MDAE$:

$$\therefore m(\angle DME) = 360^\circ - (90^\circ + 90^\circ + 120^\circ) = 60^\circ$$

$$\therefore m(\angle YMX) = m(\angle DME) = 60^\circ \quad (\text{V.O.A})$$

$$\therefore MY = MX = r$$

$\therefore \triangle XMY$ is an equilateral triangle. (Q.E.D.)

4

[a] In $\triangle AMC$: $\therefore MA = MC = r$

$$\therefore m(\angle MCA) = m(\angle MAC) = 25^\circ \quad (1)$$

In $\triangle BMC$: $\therefore MB = MC = r$

$$\therefore m(\angle MCB) = m(\angle MBC) = 45^\circ \quad (2)$$

From (1) and (2):

$$\therefore m(\angle ACB) = m(\angle MCA) + m(\angle MCB)$$

$$\therefore m(\angle ACB) = 25^\circ + 45^\circ = 70^\circ$$

$$\therefore m(\angle AMB) = 2 m(\angle ACB) = 2 \times 70^\circ = 140^\circ$$

(central and inscribed angles subtended by \widehat{AB})

(The req.)

Answers of Final Examinations

- [b] \therefore ABCE is a cyclic quadrilateral
 $\therefore m(\angle XEA) = m(\angle ABC)$
 \therefore ABDF is a cyclic quadrilateral
 $\therefore m(\angle XFA) = m(\angle ABD)$
 $\therefore m(\angle ABC) + m(\angle ABD) = 180^\circ$
 $\therefore m(\angle XEA) + m(\angle XFA) = 180^\circ$
 \therefore AFXE is a cyclic quadrilateral. (Q.E.D.)

5

- [a] $\therefore \overline{AB}$, \overline{AC} are two tangent-segments to the greater circle
 $\therefore AB = AC$
 $\therefore 2x - 3 = 15 \quad \therefore 2x = 18$
 $\therefore x = 9 \text{ cm.}$
 $\therefore \overline{AC}$, \overline{AD} are two tangent-segments to the smaller circle
 $\therefore AC = AD \quad \therefore y - 2 = 15$
 $\therefore y = 17 \text{ cm.}$ (The req.)

- [b] \therefore ABCD is a parallelogram
 $\therefore AD = BC \quad \therefore BE = AD$
 $\therefore BC = BE$
 \therefore In $\triangle BCE$: $m(\angle C) = m(\angle BEC)$
 $\therefore m(\angle C) = m(\angle BAD)$ (from the parallelogram)
 $\therefore m(\angle BAD) = m(\angle BED)$ and they are drawn on \overline{BD} and on one side of it
 \therefore The figure ABDE is a cyclic quadrilateral. (Q.E.D.)

12 Damietta

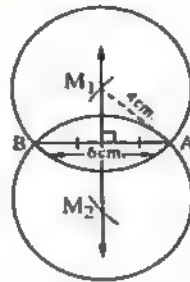
1

- 1 b 2 d 3 c 4 b 5 a 6 b

2

- [a] $\therefore \overline{AD}$ is a tangent
 $\therefore \overline{MD} \perp \overline{AD} \quad \therefore m(\angle MDA) = 90^\circ$
 $\therefore E$ is a midpoint of \overline{BC}
 $\therefore \overline{ME} \perp \overline{BC} \quad \therefore m(\angle MEA) = 90^\circ$
 From the quadrilateral ADME
 $\therefore m(\angle DME) = 360^\circ - (65^\circ + 90^\circ + 90^\circ) = 115^\circ$
 (The req.)

[b]



\therefore We can draw two circles.

3

- [a] $\therefore m(\angle BMC) = 2m(\angle A)$
 (central and inscribed angles subtended by \widehat{BC})
 $\therefore m(\angle BMC) = 2 \times 30^\circ = 60^\circ$ (First req.)
 In $\triangle MBC$: $\therefore MB = MC = r$
 $\therefore m(\angle BMC) = 60^\circ$
 $\therefore \triangle MBC$ is equilateral. (Second req.)
- [b] $\therefore \overline{AD} \parallel \overline{BC}$
 $\therefore m(\widehat{AB}) = m(\widehat{DC}) \quad \therefore AB = DC$
 $\therefore \overline{MX} \perp \overline{AB} \quad \therefore \overline{MY} \perp \overline{DC}$
 $\therefore MX = MY$ (Q.E.D.)

4

- [a] $\therefore \overline{CB}$ is a tangent
 $\therefore m(\angle BAE) = m(\angle CBE)$
 (inscribed and tangency angles subtended by \widehat{BE})
 $\therefore m(\widehat{BE}) = m(\widehat{EA})$
 $\therefore m(\angle BAE) = m(\angle EAF)$
 $\therefore m(\angle CBD) = m(\angle CAD)$ and they are drawn on \overline{CD} and on one side of it
 \therefore ABCD is a cyclic quadrilateral (Q.E.D.)
- [b] $\therefore m(\angle XYZ)$ (tangency)
 $= m(\angle L)$ (inscribed) $= 70^\circ$
 $\therefore \overline{XY}$, \overline{XZ} are two tangents
 $\therefore XY = XZ$
 $\therefore m(\angle XYZ) = m(\angle XZY) = 70^\circ$
 In $\triangle XYZ$:
 $\therefore m(\angle X) = 180^\circ - 2 \times 70^\circ = 40^\circ$ (First req.)
 In $\triangle LZY$: $\therefore YZ = LZ$
 $\therefore m(\angle LYZ) = m(\angle L) = 70^\circ$
 $\therefore m(\angle LYZ) = m(\angle XZY)$ and they are alternate angles.
 $\therefore \overline{XZ} \parallel \overline{YL}$ (Second req.)

Geometry

5

[a] In $\triangle ABC$:

$$\therefore AC = BC$$

$$\therefore m(\angle B) = m(\angle CAB) \quad (1)$$

 $\therefore \overline{AB} \parallel \overline{CD}, \overline{AC}$ is transversal

$$\therefore m(\angle DCA) = m(\angle CAB) \text{ (alternate angles)} \quad (2)$$

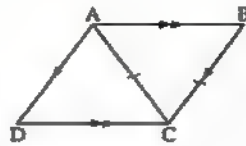
From (1) and (2): $\therefore m(\angle DCA) = m(\angle B)$
 $\therefore \overline{CD}$ is a tangent to the circle circumscribed about the triangle ABC (Q.E.D.)
[b] $\therefore LMNE$ is a cyclic quadrilateral

$$\therefore m(\angle MLN) = m(\angle MEN) = 35^\circ \quad (\text{First req.})$$

$$\therefore m(\angle ELN) = m(\angle ELM) - m(\angle MLN)$$

$$\therefore m(\angle ELN) = 80^\circ - 35^\circ = 45^\circ$$

$$\therefore m(\angle EMN) = m(\angle ELN) = 45^\circ \quad (\text{Second req.})$$



$$\therefore m(\angle ADE) = \frac{1}{2} m(\angle AME)$$

(inscribed and central angles subtended by \widehat{AE})

$$\therefore m(\angle ADB) = \frac{1}{2} \times 70^\circ = 35^\circ \quad (\text{The req.})$$

3

[a] $\therefore \overline{AD} \parallel \overline{CB}$

$$\therefore m(\widehat{BD}) = m(\widehat{AC})$$

$$\therefore m(\angle BAD) = m(\angle CDA)$$

$$\therefore \text{In } \triangle ADE: EA = ED \quad (\text{Q.E.D.})$$

[b] $\therefore \overline{EA}, \overline{EB}$ are two tangents to the circle

$$\therefore EA = EB$$

In $\triangle ABE$:

$$\therefore m(\angle EAB) = m(\angle EBA) = \frac{180^\circ - 50^\circ}{2} = 65^\circ$$

$$\therefore m(\angle ADC) \text{ (inscribed)}$$

$$= m(\angle CAE) \text{ (tangency)} = 115^\circ$$

$$\therefore m(\angle BAC) = 115^\circ - 65^\circ = 50^\circ$$

$$\therefore m(\angle AEB) = m(\angle BAC)$$

 $\therefore \overline{AC}$ is a tangent to the circle passing through the points A, B and E (Q.E.D.)

13 Kafr El-Sheikh

1

[1] a [2] c [3] c [4] b [5] b [6] d

2

[a] Construction:

Draw \overline{MC}

Proof:

$$\therefore \overline{MX} \perp \overline{BC}$$

 $\therefore X$ is the midpoint of \overline{BC}

$$\therefore XC = 8 \text{ cm.}$$

In $\triangle XMC$:

$$\therefore m(\angle CXM) = 90^\circ, CM = r = 10 \text{ cm.}$$

$$\therefore MX = \sqrt{(CM)^2 - (XC)^2} = \sqrt{100 - 64} = \sqrt{36} = 6 \text{ cm.}$$

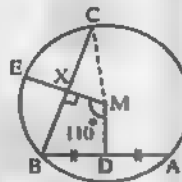
$$\therefore XE = 10 - 6 = 4 \text{ cm.} \quad (\text{First req.})$$

 $\therefore D$ is the midpoint of \overline{AB}

$$\therefore \overline{MD} \perp \overline{AB}$$

From the quadrilateral BDMX:

$$\therefore m(\angle ABC) = 360^\circ - (90^\circ + 90^\circ + 110^\circ) = 70^\circ \quad (\text{Second req.})$$

[b] $\therefore \overline{BA}$ is a tangent

$$\therefore \overline{MA} \perp \overline{AB} \quad \therefore m(\angle BAM) = 90^\circ$$

$$\text{In } \triangle AMB: m(\angle AMB) = 180^\circ - (90^\circ + 20^\circ) = 70^\circ$$

4

[a] $\therefore ABCD$ is a cyclic quadrilateral

$$\therefore m(\angle ADC) = m(\angle ABE) = 110^\circ$$

$$\therefore m(\angle ADB) = \frac{1}{2} m(\widehat{AB}) = \frac{1}{2} \times 100^\circ = 50^\circ$$

$$\therefore m(\angle BDC) = 110^\circ - 50^\circ = 60^\circ \quad (\text{The req.})$$

[b] $\therefore \overline{FB}, \overline{FD}$ are two tangents to the circle

$$\therefore BF = DF = 4 \text{ cm.}$$

$$\therefore AB = 10 + 4 = 14 \text{ cm.}$$

 $\therefore \overline{AB}, \overline{AC}$ are two tangents to the circle

$$\therefore AC = AB = 14 \text{ cm.}$$

$$\therefore EC = 14 - 9 = 5 \text{ cm.} \quad (\text{The req.})$$

5

[a] $\therefore X$ is the midpoint of \overline{AB}

$$\therefore \overline{MX} \perp \overline{AB}$$

 $\therefore Y$ is the midpoint of \overline{AC}

$$\therefore \overline{MY} \perp \overline{AC} \quad \therefore MX = MY$$

$$\therefore AB = AC$$

$$\text{In } \triangle ABC: \therefore m(\angle C) = m(\angle B) = 70^\circ$$

$$\therefore m(\angle A) = 180^\circ - (70^\circ + 70^\circ) = 40^\circ \quad (\text{The req.})$$

Answers of Final Examinations

[b] In $\triangle ADE$, $\triangle ACE$

$$\begin{cases} AD = AC \\ m(\angle DAE) = m(\angle CAE) \\ \overline{AE} \text{ is a common side} \end{cases}$$

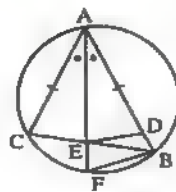
$$\therefore \triangle ADE \cong \triangle ACE$$

$$\therefore m(\angle ADE) = m(\angle ACE)$$

$$\therefore m(\angle AFB) = m(\angle ACB)$$

(two inscribed angles subtended by \widehat{AB})

$$\therefore m(\angle AFB) = m(\angle ADE)$$

 \therefore BDEF is a cyclic quadrilateral. (Q.E.D.)

El-Beheira

1

1 d 2 c 3 b 4 b 5 c 6 a

2

[a] \therefore X is the midpoint of \overline{AC}

$$\therefore \overline{MX} \perp \overline{AC} \quad \therefore m(\angle AXY) = 90^\circ$$

 $\therefore \overline{YB}$ is a tangent to the circle

$$\therefore \overline{MB} \perp \overline{BY} \quad \therefore m(\angle MBY) = 90^\circ$$

 $\therefore m(\angle AXY) = m(\angle ABY)$ and they are drawn on \overline{AY} and on one side of it \therefore AXBY is a cyclic quadrilateral. (Q.E.D.)[b] $\therefore \overline{CM} \parallel \overline{AB}$, \overline{AM} is a transversal

$$\therefore m(\angle CMA) = m(\angle A) = 60^\circ$$

$$\therefore m(\angle B) = \frac{1}{2} m(\angle CMA)$$

(two inscribed angles subtended by \widehat{AC})

$$\therefore m(\angle B) = \frac{1}{2} \times 60^\circ = 30^\circ \quad (\text{The req.})$$

3

[a] $\therefore m(\angle B) = m(\angle C) \quad \therefore AB = AC$ \therefore X is the midpoint of \overline{AB}

$$\therefore \overline{MX} \perp \overline{AC}, \overline{MY} \perp \overline{AC}$$

$$\therefore MX = MY \quad (\text{Q.E.D.})$$

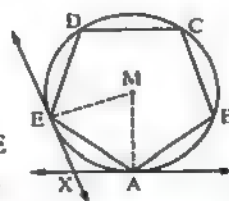
[b] Construction :

Draw \overline{AM} , \overline{ME}

Proof :

$$\therefore AB = BC = CD = DE = AE$$

(The properties of the regular pentagon)



$$\therefore m(\widehat{AB}) = m(\widehat{BC}) = m(\widehat{CD}) = m(\widehat{DE}) = m(\widehat{AE})$$

$$\therefore \text{measure of the circle} = 360^\circ$$

$$\therefore m(\widehat{AE}) = \frac{360^\circ}{5} = 72^\circ \quad (\text{First req.})$$

$$\therefore m(\angle AME) = m(\widehat{AE}) = 72^\circ$$

 $\therefore \overline{AX}$ is a tangent to the circle at A

$$\therefore m(\angle MAX) = 90^\circ$$

similarly $m(\angle MEX) = 90^\circ$

In the quadrilateral MAXE :

$$\therefore m(\angle AXE) = 360^\circ - (72^\circ + 90^\circ + 90^\circ) = 108^\circ \quad (\text{Second req.})$$

4

[a] In $\triangle AMC$: $\therefore AM = MC = r$

$$\therefore m(\angle MAC) = m(\angle ACM)$$

$$\therefore m(\angle BAC) = m(\angle MAC)$$

 $\therefore m(\angle BAC) = m(\angle ACM)$ and they are alternate angles.

$$\therefore \overline{AB} \parallel \overline{CM}$$

 \therefore D is the midpoint of \overline{AB}

$$\therefore \overline{MD} \perp \overline{AB}$$

$$\therefore \overline{DM} \perp \overline{CM}$$

$$\therefore \overline{AB} \parallel \overline{CM}$$

(Q.E.D.)

[b] $\therefore \overline{AC}$ is a tangent to the circle M at A

$$\therefore \overline{MA} \perp \overline{AC}$$

$$\therefore m(\angle CAM) = 90^\circ$$

 $\therefore \overline{BD}$ is a tangent to the circle M at B

$$\therefore \overline{MB} \perp \overline{BD}$$

$$\therefore m(\angle EBM) = 90^\circ$$

In $\triangle CAM$, $\triangle EBM$:

$$\begin{cases} m(\angle CAM) = m(\angle EBM) = 90^\circ \\ m(\angle AMC) = m(\angle BME) \text{ (V.O.A.)} \\ MA = MB \text{ (lengths of two radii)} \end{cases}$$

$$\therefore \text{The two triangles are congruent and we deduce that } CM = EM$$

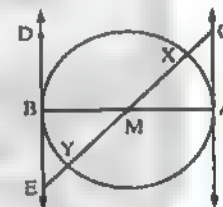
$$\therefore XM = YM \text{ (lengths of two radii)}$$

 \therefore The two triangles are congruent and we deduce that $CM = EM$

$$\therefore XM = YM \text{ (lengths of two radii)}$$

 \therefore by subtracting

$$\therefore CX = YE \quad (\text{Q.E.D.})$$



5

[a] $\therefore \overline{XA}$, \overline{XB} are two tangents to the circle

$$\therefore XA = XB$$

Geometry

∴ In $\triangle ABX$

$$m(\angle XAB) \approx m(\angle XBA) = \frac{180^\circ - 50^\circ}{2} = 65^\circ$$

∴ ABCD is a cyclic quadrilateral

$$m(\angle BAD) + m(\angle DCB) = 180^\circ$$

$$\therefore m(\angle BAD) = 180^\circ - 115^\circ = 65^\circ$$

$$\therefore m(\angle XAB) = m(\angle BAD)$$

$$\therefore \overline{AB} \text{ bisects } \angle DAX \quad (\text{Q.E.D.1})$$

$$\begin{aligned} \therefore m(\angle ADB) (\text{inscribed}) \\ = m(\angle XAB) (\text{tangency}) = 65^\circ \end{aligned}$$

$$\therefore m(\angle BAD) = m(\angle ADB)$$

$$\therefore BD = BA \quad (\text{Q.E.D.2})$$

[b] ∴ $AB = CD$

$$\therefore m(\widehat{AB}) = m(\widehat{CD})$$

Subtracting $m(\widehat{BD})$ from both sides

$$\therefore m(\widehat{AD}) = m(\widehat{BC})$$

$$\therefore m(\angle ACD) = m(\angle BAC)$$

$$\therefore \text{In } \triangle ACE : AE = CE$$

$$\therefore \triangle ACE \text{ is an isosceles triangle.} \quad (\text{Q.E.D.})$$

15 El-Fayoum

1

- 1 c 2 b 3 a 4 d 5 a 6 c

2

[a] ∴ $AB = CD$

$$\therefore \overline{ME} \perp \overline{AB}, \overline{MO} \perp \overline{CD}$$

$$\therefore ME = MO \quad \therefore X + 2 = 6$$

$$\therefore X = 4 \text{ cm.} \quad (\text{First req.})$$

$$\therefore CD = AB = 3 \times 4 + 4 = 16 \text{ cm.} \quad (\text{Second req.})$$

[b] ∴ $m(\angle C) = \frac{1}{2} m(\angle AMB)$

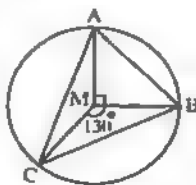
$$= \frac{1}{2} \times 90^\circ = 45^\circ$$

(inscribed and central angles subtended by \widehat{AB})

$$\therefore m(\angle A) = \frac{1}{2} m(\angle BMC) = \frac{1}{2} \times 130^\circ = 65^\circ$$

(inscribed and central angles subtended by \widehat{BC})

$$\therefore m(\angle B) = 180^\circ - (45^\circ + 65^\circ) = 70^\circ \quad (\text{The req.})$$



3

[a] Construction :

Draw \overline{MB}

Proof :

∴ \overline{AB} is a tangent to the circle

$$\therefore \overline{MB} \perp \overline{AB}$$

$$\therefore m(\angle MBA) = 90^\circ$$

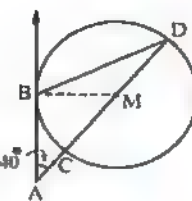
In $\triangle ABM$:

$$m(\angle BMA) = 180^\circ - (90^\circ + 40^\circ) = 50^\circ$$

$$m(\angle BDC) = \frac{1}{2} m(\angle BMC) = \frac{1}{2} \times 50^\circ = 25^\circ$$

(inscribed and central angles subtended by \widehat{BC})

(The req.)



[b] ∴ X is the midpoint of \overline{AC}

$$\therefore \overline{MX} \perp \overline{AC} \quad \therefore m(\angle AXM) = 90^\circ$$

∴ \overline{YB} is a tangent to the circle

$$\therefore \overline{MB} \perp \overline{BY} \quad \therefore m(\angle MBY) = 90^\circ$$

∴ $m(\angle AXM) = m(\angle MBY)$ and they are drawn on \overline{AY} and on one side of it

$$\therefore AXBY \text{ is a cyclic quadrilateral} \quad (\text{Q.E.D.})$$

4

[a] Construction :

Draw $\overline{XM}, \overline{YM}, \overline{ZM}$

$\overline{AY}, \overline{CM}$

Proof :

$$\therefore \overline{XM} \perp \overline{AB}, \overline{YM} \perp \overline{BC}$$

$$\therefore \overline{ZM} \perp \overline{AC}$$

$$\therefore XM = YM = ZM = r$$

$$\therefore AB = BC = AC$$

∴ $\triangle ABC$ is an equilateral triangle (First req.)

In $\triangle MYC$: $m(\angle MYC) = 90^\circ$

$$\therefore (YC)^2 = (MC)^2 - (MY)^2 = (4)^2 - (2)^2 = 12$$

$$\therefore YC = 2\sqrt{3} \text{ cm.} \quad \therefore BC = 4\sqrt{3} \text{ cm.}$$

$$\therefore \text{The area of } \triangle ABC = \frac{1}{2} \times BC \times AY$$

$$= \frac{1}{2} \times 4\sqrt{3} \times 6$$

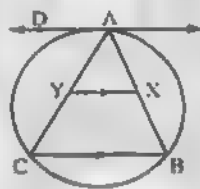
$$= 12\sqrt{3} \text{ cm}^2. \quad (\text{Second req.})$$



- [b] $\therefore m(\angle BCD) = \frac{1}{2} m(\angle BMD)$
 (inscribed and central angles subtended by \widehat{BD})
 $\therefore m(\angle BCD) = \frac{1}{2} \times 130^\circ = 65^\circ$
 $\therefore \overline{AB} \parallel \overline{CD}$, \overline{BC} is a transversal
 $\therefore m(\angle ABC) = m(\angle BCD) = 65^\circ$
 (alternate angles)
 $\therefore \overline{AB}$, \overline{AC} are two tangent segments
 $\therefore AB = AC$
 $\therefore m(\angle ACB) = m(\angle ABC) = 65^\circ$ (2)
 From (1) and (2):
 $\therefore m(\angle ACB) = m(\angle BCD) = 65^\circ$
 $\therefore \overline{CB}$ bisects $\angle ACD$ (Q.E.D.)

5

- [a] $\therefore \overline{AD}$ is a tangent to the circle
 $\therefore m(\angle DAC)$ (tangency)
 $= m(\angle B)$ (inscribed) (1)
 $\therefore \overline{XY} \parallel \overline{BC}$, \overline{AB}
 is a transversal
 $\therefore m(\angle AXY) = m(\angle B)$ (2)
 (corresponding angles)
 From (1) and (2): $\therefore m(\angle AXY) = m(\angle DAC)$
 $\therefore \overline{AD}$ is a tangent to the circle passing through
 the points A, X and Y (Q.E.D.)
- [b] $\therefore X$ is the midpoint of \overline{AC}
 $\therefore \overline{MX} \perp \overline{AC}$
 $\therefore m(\angle CXM) = 90^\circ$
 $\therefore \overline{BD}$ is a tangent to the circle
 $\therefore \overline{BD} \perp \overline{AB}$
 $\therefore m(\angle DBM) = 90^\circ$
 $\therefore m(\angle CXM) + m(\angle DBM) = 180^\circ$
 $\therefore XMBD$ is a cyclic quadrilateral (Q.E.D.1)
 $\therefore \angle BMY$ is an exterior angle of the cyclic
 quadrilateral XMBD
 $\therefore m(\angle BMY) = m(\angle D)$ (1)
 $\therefore m(\angle BAY) = \frac{1}{2} m(\angle BMY)$ (2)
 (inscribed and central angles subtended
 the same arc \widehat{BY})
 From (1) and (2):
 $\therefore m(\angle BAY) = \frac{1}{2} m(\angle D)$ (Q.E.D.2)



16 Beni Suef

1

- 1 c 2 a 3 c 4 c 5 b 6 c

2

- [a] $\therefore m(\angle AMB) = 2 m(\angle ADB) = 2 \times 70^\circ = 140^\circ$
 (central and inscribed angles subtended by \widehat{AB})
 In $\triangle ABM$: $\therefore \overline{MC} \perp \overline{AB}$
 $\therefore MA = MB = r$
 $\therefore m(\angle AMC) = \frac{1}{2} m(\angle AMB) = \frac{1}{2} \times 140 = 70$
 (The req.)
- [b] $\therefore AB = CD$
 $\therefore \overline{MX} \perp \overline{AB}$, $\overline{NY} \perp \overline{CD}$
 $\therefore MX = NY$, $\overline{MX} \parallel \overline{NY}$
 $\therefore MXYN$ is a rectangle (Q.E.D.)

3

- [a] $\therefore D$ is the midpoint of \overline{AB}
 $\therefore \overline{MD} \perp \overline{AB}$ $\therefore m(\angle ADM) = 90^\circ$
 $\therefore E$ is the midpoint of \overline{AC}
 $\therefore \overline{ME} \perp \overline{AC}$ $\therefore m(\angle AEM) = 90^\circ$
 $\therefore ADME$ is a cyclic quadrilateral
 $\therefore m(\angle DME) = 360^\circ - (90^\circ + 90^\circ + 50^\circ) = 130^\circ$
 (The req.)
- [b] $\therefore AB = BC$
 $\therefore m(\angle BAC) = m(\angle ACB) = 55^\circ$
 $\therefore m(\angle BDC) = m(\angle BAC) = 55^\circ$ and they are
 drawn on \overline{BC} and on one side of it
 $\therefore ABCD$ is a cyclic quadrilateral (Q.E.D.)

4

- [a] $\therefore m(\angle ACB) = \frac{1}{2} m(\angle AMB)$
 (inscribed and central angles subtended the same
 arc \widehat{AB})
 $\therefore m(\angle ACB) = \frac{1}{2} \times 120^\circ = 60^\circ$ (1)
 $\therefore \overline{ED} \parallel \overline{AB}$
 $\therefore m(\widehat{AC}) = m(\widehat{BC})$
 $\therefore AC = BC$ (2)
 From (1) and (2):
 $\therefore \triangle CAB$ is an equilateral triangle. (Q.E.D.)

Geometry

[b] Construction :

Draw \overline{BC}

Proof :

$\therefore \overline{AB}, \overline{AC}$ are two
tangents to the circle

$$\therefore AB = AC$$

$$\therefore m(\angle ABC) = m(\angle ACB)$$

$$\therefore m(\angle ABC) \text{ (tangency)} \\ = m(\angle BDC) \text{ (inscribed)} = 70^\circ$$

\therefore In $\triangle ABC$:

$$m(\angle A) = 180^\circ - (70^\circ + 70^\circ) = 40^\circ \quad (\text{The req.})$$

5

[a] $\therefore \overline{AB}, \overline{AC}$ are two tangent-segments to the circle

$$\therefore AB = AC$$

$$\therefore m(\angle ABC) = m(\angle ACB) \quad (1)$$

In $\triangle BCD$: $\therefore BC = BD$

$$\therefore m(\angle BDC) = m(\angle BCD) \quad (2)$$

$$\therefore m(\angle BDC) \text{ (inscribed)} \\ = m(\angle ABC) \text{ (tangency)} \quad (3)$$

From (1), (2) and (3) :

$$\therefore m(\angle A) = m(\angle CBD)$$

$\therefore \overline{BD}$ is a tangent to the circle passing through
the vertices of $\triangle ABC$ (Q.E.D.)

[b] $\therefore \overline{BC}$ is a tangent to the circle

$$\therefore \overline{AB} \perp \overline{BC}$$

$$\therefore m(\angle ABC) = 90^\circ$$

$\therefore E$ is the midpoint of \overline{AD}

$$\therefore \overline{ME} \perp \overline{AD}$$

$$\therefore m(\angle CEM) = 90^\circ$$

$$\therefore m(\angle ABC) + m(\angle CEM) = 180^\circ$$

$\therefore EMBC$ is a cyclic quadrilateral (Q.E.D.)

17 El-Menia

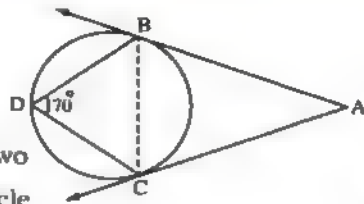
1

1. b 2. d 3. b 4. b 5. c 6. a

2

[a] $\therefore X$ is the midpoint of \overline{AB}

$$\therefore \overline{MX} \perp \overline{AB}$$



$\therefore Y$ is the midpoint of \overline{AC}

$$\therefore \overline{MY} \perp \overline{AC}$$

$$\therefore AB = AC$$

$$\therefore MX = MY$$

$$\therefore ME = MD = r$$

$$\therefore XE = YD$$

(Q.E.D.)

[b] In $\triangle ABC$: $\therefore AB = AD$

$$\therefore m(\angle ABD) = m(\angle ADB) = 30^\circ$$

$$\therefore m(\angle A) = 180^\circ - 2 \times 30^\circ = 120^\circ$$

$$\therefore m(\angle A) + m(\angle C) = 120^\circ + 60^\circ = 180^\circ$$

$\therefore ABCD$ is a cyclic quadrilateral (Q.E.D.)

3

[a] Construction :

Draw \overline{AM}

Proof :

$$\therefore \overline{MD} \perp \overline{AB}$$

$$\therefore m(\angle MDB) = 90^\circ$$

$\therefore X$ is the midpoint of \overline{BC}

$$\therefore \overline{MX} \perp \overline{BC}$$

$$\therefore m(\angle MXB) = 90^\circ$$

In the quadrilateral $MDXB$:

$$\therefore m(\angle DMX) = 360^\circ - (56^\circ + 90^\circ + 90^\circ) = 124^\circ$$

(First req.)

$$\therefore \overline{MD} \perp \overline{AB}$$

$\therefore D$ is the midpoint of \overline{AB}

$$\therefore AD = 4 \text{ cm.}$$

In $\triangle ADM$:

$$(MD)^2 = (AM)^2 - (AD)^2 = (5)^2 - (4)^2 = 25 - 16 = 9$$

$$\therefore MD = 3 \text{ cm.}$$

$$\therefore DE = 5 - 3 = 2 \text{ cm.}$$

(Second req.)

[b] $\therefore \overline{AD}$ is a tangent to the circle

$$\therefore m(\angle DAB) \text{ (tangency)}$$

$$= m(\angle ACB) \text{ (inscribed)} \quad (1)$$

$\therefore \overline{XY} \parallel \overline{BC}$, \overline{YC} is a transversal

$$\therefore m(\angle AYX) = m(\angle ACB)$$

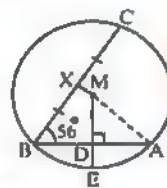
(corresponding angles)

(2)

\therefore From (1) and (2) :

$$\therefore m(\angle DAB) = m(\angle AYX)$$

$\therefore \overline{AD}$ is a tangent to the circle passing through
the points A, X and Y (Q.E.D.)



Answers of Final Examinations

4

- [a] $\because AB = AC$
 $\therefore m(\widehat{AB}) = m(\widehat{AC})$
 $\therefore m(\angle AEB) = m(\angle AEC)$ (Q.E.D.)
- [b] $\because \overline{XA}, \overline{XB}$ are two tangents to the circle
 $\therefore XA = XB$
 $\therefore m(\angle XAB) = m(\angle XBA) = \frac{180^\circ - 70^\circ}{2} = 55^\circ$ (1)
 $\because ABCD$ is a cyclic quadrilateral
 $\therefore m(\angle DAB) = 180^\circ - 125^\circ = 55^\circ$ (2)
 From (1) and (2):
 $\therefore m(\angle DAB) = m(\angle XAB)$ (Q.E.D.)

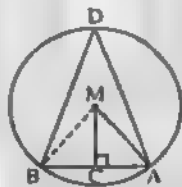
5

[a] Construction :

Draw \overline{MB}

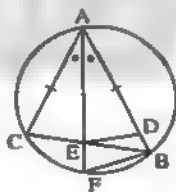
Proof :

- $\because MA = MB = r$
 $\therefore \overline{MC} \perp \overline{AB}$
 $\therefore \overline{MC}$ bisects $\angle AMB$
 $\therefore m(\angle AMC) = \frac{1}{2} m(\angle AMB)$ (1)
 $\because m(\angle ADB) = \frac{1}{2} m(\angle AMB)$ (2)
 (inscribed and central angles subtended by \widehat{AB})
 \therefore From (1) and (2):
 $\therefore m(\angle AMC) = m(\angle ADB)$ (Q.E.D.)

[b] \because In $\triangle ADE, \triangle ACE$

- $\begin{cases} AD = AC \\ m(\angle DAE) = m(\angle CAE) \\ \overline{AE} \text{ is a common side} \end{cases}$

- $\therefore \triangle ADE \cong \triangle ACE$
 $\therefore m(\angle ADE) = m(\angle ACE)$
 $\because m(\angle AFB) = m(\angle ACB)$
 (two inscribed angles subtended by \widehat{AB})
 $\therefore m(\angle AFB) = m(\angle ADE)$
 $\therefore BDEF$ is a cyclic quadrilateral. (Q.E.D.)



2

- [a] $\because \overline{MN}$ is the line of centres
 $\therefore \overline{AB}$ is the common chord.
 $\therefore \overline{AB} \perp \overline{MN}$ $\therefore m(\angle BEN) = 90^\circ$
 In the quadrilateral CDNE:
 $\therefore m(\angle CDN) = 360^\circ - (140^\circ + 40^\circ + 90^\circ) = 90^\circ$
 $\therefore \overline{ND} \perp \overline{CD}$
 $\therefore \overline{CD}$ is a tangent to the circle N at D (Q.E.D.)
- [b] $\because AB = CD$ (properties of the rectangle)
 $\therefore CE = CD$ $\therefore AB = CE$
 $\therefore m(\widehat{AB}) = m(\widehat{CE})$ and adding $m(\widehat{BE})$
 to both sides.
 $\therefore m(\widehat{AE}) = m(\widehat{BC})$
 $\therefore AE = BC$ (Q.E.D.)

3

- [a] State by yourself.
- [b] $\because \overline{XY}, \overline{XZ}$ are two tangents to the circle
 $\therefore XY = XZ$
 \therefore In $\triangle XYZ$:
 $m(\angle XYZ) = m(\angle XZY) = \frac{180^\circ - 50^\circ}{2} = 65^\circ$
 $\because YZDE$ is a cyclic quadrilateral
 $\therefore m(\angle EYZ) + m(\angle D) = 180^\circ$
 $\therefore m(\angle EYZ) = 180^\circ - 115^\circ = 65^\circ$
 $\because m(\angle YEZ)$ (inscribed)
 $= m(\angle XYZ)$ (tangency) $= 65^\circ$
 $\therefore m(\angle EYZ) = m(\angle YEZ)$
 \therefore In $\triangle YZE: ZE = ZY$ (Q.E.D.)

4

- [a] In $\triangle ABC: \because m(\angle B) = m(\angle C)$
 $\therefore AB = AC$
 $\therefore X$ is the midpoint of \overline{AB}
 $\therefore \overline{MX} \perp \overline{AB}$ $\therefore \overline{MY} \perp \overline{AC}$
 $\therefore MX = MY$ (Q.E.D.)
- [b] $\because \overline{XY}$ is a tangent to the circle
 $\therefore \overline{MY} \perp \overline{XY}$ $\therefore m(\angle XMY) = 90^\circ$
 In $\triangle XMY$:
 $\therefore m(\angle XMY) = 180^\circ - (90^\circ + 40^\circ) = 50^\circ$

18 Assiut

1

- [1] c [2] d [3] b [4] c [5] d [6] b

Geometry

$$\begin{aligned} \therefore m(\angle YDC) &= \frac{1}{2} m(\angle YMC) \\ (\text{inscribed and central angles subtended by } \widehat{YC}) \\ \therefore m(\angle YDC) &= \frac{1}{2} \times 50^\circ = 25^\circ \quad (\text{The req.}) \end{aligned}$$

5

[a] In $\triangle ABC$: $\therefore CB = AC$

$$\therefore m(\angle BAC) = m(\angle ABC) = 65^\circ$$

$$\therefore m(\angle CAD) = 130^\circ - 65^\circ = 65^\circ$$

$$\therefore m(\angle ABC) = m(\angle CAD) = 65^\circ$$

$\therefore \overline{AD}$ is a tangent to the circle passing through the vertices of the triangle ABC (Q.E.D.)

[b] $\therefore \overline{XY} \parallel \overline{BD}$, \overline{AB} is a transversal

$$\therefore m(\angle DBX) = m(\angle YXB) \quad (1)$$

(alternate angles)

$$\therefore m(\angle C) (\text{inscribed}) = m(\angle ABD) (\text{tangency}) \quad (2)$$

From (1) and (2) :

$$\therefore m(\angle C) = m(\angle YXB)$$

$\therefore AXYC$ is a cyclic quadrilateral. (Q.E.D.)

19 Souhag

1

1 b 2 c 3 d 4 c 5 b 6 b

2

[a] $\therefore m(\angle AMB) = 90^\circ \therefore m(\widehat{AB}) = 90^\circ$

$$\therefore r = 7 \text{ cm.}$$

$$\therefore \text{The length of } \widehat{AB} = \frac{90^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 7 = 11 \text{ cm.} \quad (\text{The req.})$$

[b] $\therefore \overline{AB}$ is a tangent

$$\therefore \overline{MA} \perp \overline{AB} \therefore m(\angle MAB) = 90^\circ$$

$\therefore E$ is the midpoint of \overline{DC}

$$\therefore \overline{ME} \perp \overline{DC} \therefore m(\angle MEB) = 90^\circ$$

From the quadrilateral ABEM :

$$\therefore m(\angle EMA) = 360^\circ - (50^\circ + 90^\circ + 90^\circ) = 130^\circ \quad (\text{The req.})$$

3

[a] State by yourself.

[b] $\therefore \angle CBE$ is an exterior angle of the cyclic quadrilateral ABCD

$$\therefore m(\angle ADC) = m(\angle CBE) = 85^\circ$$

$$\begin{aligned} \therefore m(\angle ADB) (\text{inscribed}) &= \frac{1}{2} m(\widehat{AB}) \\ &= \frac{1}{2} \times 110^\circ = 55^\circ \end{aligned}$$

$$\therefore m(\angle BDC) = 85^\circ - 55^\circ = 30^\circ \quad (\text{The req.})$$

4

[a] $\therefore \overline{AB}$, \overline{CD} are two tangents to the circles M, N

In circle M

$$BF = DF \quad (1)$$

$$\therefore \text{in circle N : } AF = CF \quad (2)$$

Subtracting (1) from (2) :

$$\therefore AF - BF = CF - DF$$

$$\therefore AB = CD \quad (\text{Q.E.D.})$$

[b] $\therefore \overline{AB}$ is a tangent to the circle

$$\therefore \overline{MB} \perp \overline{AB} \therefore m(\angle ABM) = 90^\circ$$

In $\triangle ABM$:

$$\therefore m(\angle AMB) = 180^\circ - (40^\circ + 90^\circ) = 50^\circ$$

$$\therefore m(\angle BDC) = \frac{1}{2} m(\angle BMC)$$

(inscribed and central angles subtended by \widehat{BC})

$$\therefore m(\angle BDC) = \frac{1}{2} \times 50^\circ = 25^\circ \quad (\text{The req.})$$

5

[a] $\therefore AB = CD$, $\overline{ME} \perp \overline{AB}$, $\overline{MF} \perp \overline{CD}$

$$\therefore ME = MF \therefore X + 2 = 6$$

$$\therefore X = 4 \text{ cm.} \quad (\text{First req.})$$

$$\therefore \overline{CD} = 3 \times 4 + 4 = 16 \text{ cm.} \quad (\text{Second req.})$$

[b] $\therefore \overline{XY} \parallel \overline{BD}$, \overline{AB} is a transversal

$$\therefore m(\angle DBX) = m(\angle BXY) \quad (1)$$

(alternate angles)

$$\therefore m(\angle C) (\text{inscribed}) = m(\angle ABD) (\text{tangency}) \quad (2)$$

From (1) and (2) :

$$\therefore m(\angle C) = m(\angle BXY)$$

$\therefore AXYC$ is a cyclic quadrilateral. (Q.E.D.)

20 Qena

1

- 1 b 2 a 3 c 4 a 5 b 6 d

2

[a] The measure of the arc $= 45^\circ \times 2 = 90^\circ$

$$\therefore \text{its length} = \frac{90^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 7$$

$$= 11 \text{ cm.}$$

(The req.)

[b] $\therefore \overline{DB}, \overline{DA}$ are two tangent to the circle M

$$\therefore DB = DA \quad (1)$$

 $\therefore \overline{DC}, \overline{DA}$ are two tangent to the circle N

$$\therefore DC = DA \quad (2)$$

From (1) and (2) : $\therefore DB = DC$ (Q.E.D.)

3

[a] Construction :

Draw \overline{CD}

Proof :

 $\therefore D$ is the midpoint of \widehat{AC}

$$\therefore m(\widehat{AD}) = m(\widehat{DC}) = 40^\circ$$

 $\therefore \overline{AB}$ is a diameter

$$\therefore m(\widehat{BC}) = 180^\circ - (40^\circ + 40^\circ) = 100^\circ$$

$$\therefore m(\angle DAB) = \frac{1}{2} m(\widehat{BD}) = \frac{1}{2} (100^\circ + 40^\circ)$$

$$= \frac{1}{2} \times 140^\circ$$

$$= 70^\circ \quad (\text{First req.})$$

$$\therefore m(\angle DCB) = \frac{1}{2} m(\widehat{BAD}) = \frac{1}{2} (180^\circ + 40^\circ)$$

$$= \frac{1}{2} \times 220^\circ = 110^\circ$$

(Second req.)

[b] $\therefore \overline{AB}, \overline{AC}$ are two chords in the circle. $\therefore X$ and Y are the two midpoints of \overline{AB} and \overline{AC}

$$\therefore \overline{MX} \perp \overline{AB}, \overline{MY} \perp \overline{AC}$$

$$\therefore m(\angle MXA) = 90^\circ, m(\angle MYA) = 90^\circ$$

In $\triangle MDE$: $\therefore DE = MD = ME = r$

$$\therefore m(\angle EMD) = 60^\circ$$

$$\therefore m(\angle XMY) = m(\angle EMD) = 60^\circ \quad (\text{V.O.A.})$$

In the quadrilateral $AXMY$:

$$\therefore m(\angle BAC) = 360^\circ - (90^\circ + 90^\circ + 60^\circ) = 120^\circ$$

(The req.)

4

[a] $\therefore \overline{AB}$ is a diameter of the circle.

$$\therefore m(\angle ACB) = 90^\circ$$

$$\therefore m(\angle ACE) = m(\angle ADE)$$

and they are drawn on \overline{AE} and on one side of it $\therefore ACDE$ is a cyclic quadrilateral. (Q.E.D.)

[b] Construction :

Draw $\overline{MX}, \overline{MY}$

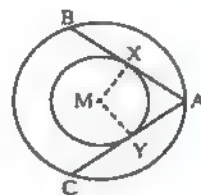
Proof :

 $\therefore \overline{AB}, \overline{AC}$ are two tangents to the smaller circle.

$$\therefore \overline{MX} \perp \overline{AB}, \overline{MY} \perp \overline{AC}$$

 $\therefore MX = MY = r$ (radii of the smaller circle)

$$\therefore AB = AC \quad (\text{Q.E.D.})$$



5

[a] $\therefore ABCD$ is a cyclic quadrilateral

$$\therefore m(\angle BAD) = 180^\circ - 70^\circ = 110^\circ$$

 $\therefore ABFE$ is a cyclic quadrilateral and $\angle BAD$ is exterior of it.

$$\therefore m(\angle EFB) = m(\angle BAD) = 110^\circ \quad (\text{First req.})$$

$$\therefore m(\angle EFB) + m(\angle BCD) = 110^\circ + 70^\circ = 180^\circ$$

and they are interior angle in the same side of \overline{FC}

$$\therefore \overline{CD} \parallel \overline{EF} \quad (\text{Second req.})$$

[b] $\therefore \overline{AB}, \overline{AC}$ are tangent-segments to the circle

$$\therefore AB = AC$$

$$\therefore m(\angle ACB) = \frac{180^\circ - 60^\circ}{2} = 60^\circ \quad (1)$$

$$\therefore m(\angle BEC) \text{ (inscribed)} \\ = m(\angle ACB) \text{ (tangency)} = 60^\circ \quad (2)$$

 $\therefore EBCD$ is cyclic quadrilateral

$$\therefore m(\angle EBC) = 180^\circ - 120^\circ = 60^\circ \quad (3)$$

 \therefore From (2) \therefore (3) in $\triangle EBC$:

$$\therefore m(\angle BCE) = 60^\circ$$

$$\therefore \triangle BCE \text{ is equilateral} \quad (\text{Q.E.D. 1})$$

From (1) \therefore (3) : $\therefore m(\angle ACB) = m(\angle EBC)$ and they are alternate angles

$$\therefore \overline{AC} \parallel \overline{BE} \quad (\text{Q.E.D. 2})$$

Geometry

21 Luxor

1

- 1 b 2 c 3 c 4 a 5 d 6 b

2

[a] $\because AB = CD$ $\therefore \overline{MH} \perp \overline{AB}, \overline{ME} \perp \overline{CD}$ $\therefore MH = ME \quad \therefore x + 2 = 6$ $\therefore x = 4 \text{ cm.} \quad (\text{First req.})$ $\therefore AB = CD = 3 \times 4 + 4 = 16 \text{ cm.} \quad (\text{Second req.})$ [b] $\because \overline{AM} \parallel \overline{CD}, \overline{MD}$ is a transversal. $\therefore m(\angle CDM) + m(\angle AMD) = 180^\circ$

(two interior angles in the same side of the transversal)

 $\therefore m(\angle CDM) = 180^\circ - 90^\circ = 90^\circ$ $\because MD = \frac{1}{2} MB \quad \therefore MC = MB = r$ $\therefore MD = \frac{1}{2} MC \quad \therefore m(\angle MCD) = 30^\circ$ $\because \overline{AM} \parallel \overline{CD}, \overline{CM}$ is a transversal. $\therefore m(\angle AMC) = m(\angle MCD) = 30^\circ$

(alternate angles)

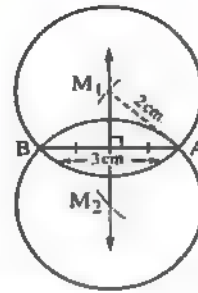
 $\therefore m(\widehat{AC}) = m(\angle AMC) = 30^\circ \quad (\text{The req.})$

3

[a] $\because \overline{AB}, \overline{AC}$ are two tangent segments $\therefore AB = AC$ $\therefore m(\angle ACB) = m(\angle ABC) = \frac{180^\circ - 50^\circ}{2} = 65^\circ$
(First req.) $\because \overline{MC}$ is a radius $\therefore \overline{MC} \perp \overline{AC}$ $\therefore m(\angle ACM) = 90^\circ$ $\therefore m(\angle BCM) = 90^\circ - 65^\circ = 25^\circ \quad (\text{Second req.})$ [b] $\because m(\widehat{AX}) = m(\widehat{AY})$ $\therefore m(\angle ACX) = m(\angle ABY)$ \because They are drawn on \overline{HD} and on one side of it. $\therefore DBCH$ is a cyclic quadrilateral. (Q.E.D.1) $\therefore m(\angle DHB) = m(\angle DCB)$ $\because m(\angle XCB) = m(\angle XAB)$ (two inscribed angles subtended by \widehat{XB}) $\therefore m(\angle DHB) = m(\angle XAB) \quad (\text{Q.E.D.2})$

4

[a]

 \therefore There are two solutions.[b] $\because \overline{BD} \parallel \overline{XY} \quad \therefore m(\widehat{BC}) = m(\widehat{CD})$ $\therefore m(\angle BAC) = m(\angle DAC) \quad (1)$ $\therefore \overline{AC}$ bisects $\angle BAD \quad (\text{Q.E.D.1})$ $\because m(\angle CBD) = m(\angle DAC) \quad (2)$ (inscribed angles subtended by \widehat{CD}) $\therefore m(\angle CBH) = m(\angle BAH)$ $\therefore \overline{BC}$ is a tangent to the circle passing by the vertices of $\triangle ABH \quad (\text{Q.E.D.2})$

5

[a] $\because \overline{AB} \parallel \overline{DC}, \overline{AD}$ is a transversal to them. $\therefore m(\angle A) + m(\angle D) = 180^\circ \quad (1)$ but $\angle CEH$ is an exterior angle of the cyclic quadrilateral $ABEH$ $\therefore m(\angle CEH) = m(\angle A) \quad (2)$

From (1) and (2):

 $\therefore m(\angle CEH) + m(\angle D) = 180^\circ$ $\therefore HDCE$ is a cyclic quadrilateral. (Q.E.D.)[b] $\because m(\widehat{BD} \text{ The major}) = 2 m(\angle BCD)$

$$= 2 \times 100^\circ = 200^\circ$$

 $\therefore m(\widehat{BCD}) = 360^\circ - 200^\circ = 160^\circ$ $\because m(\widehat{HE}) = m(\angle HME) = 50^\circ$ $\therefore m(\angle A) = \frac{1}{2} [m(\widehat{BCD}) - m(\widehat{HE})]$
 $= \frac{1}{2} [160^\circ - 50^\circ] = 55^\circ \quad (\text{The req.})$

22 Aswan

1

- 1 d 2 b 3 a 4 c 5 b 6 c

Answers of Final Examinations

2

[a] $\because \overline{AB}$ is a tangent to the circle.

$$\therefore \overline{MA} \perp \overline{AB} \quad \therefore m(\angle MAB) = 90^\circ$$

In $\triangle ABM$:

$$\therefore (BM)^2 = (AB)^2 + (AM)^2 = (8)^2 + (6)^2 = 100$$

$$\therefore BM = 10 \text{ cm.}$$

$$\because MA = MD = 6 \text{ cm.}$$

$$\therefore BD = 10 - 6 = 4 \text{ cm.} \quad (\text{The req.})$$

[b] $\because ABCD$ is a cyclic quadrilateral.

$$\therefore m(\angle BCD) + m(\angle BAD) = 180^\circ$$

$$\therefore m(\angle BCD) = 180^\circ - 120^\circ = 60^\circ \quad (\text{First req.})$$

 $\because \overline{BF} \parallel \overline{DC}$, \overline{BC} is a transversal.

$$\therefore m(\angle CBF) = m(\angle BCD) = 60^\circ$$

(alternate angles)

$$\therefore m(\angle CBE) = 60^\circ + 55^\circ = 115^\circ$$

 $\because \angle CBE$ is an exterior angle of a cyclic quadrilateral.

$$\therefore m(\angle ADC) = m(\angle CBE) = 115^\circ \quad (\text{Second req.})$$

3

[a] $\because D$ is midpoint of \overline{AB}

$$\therefore \overline{MD} \perp \overline{AB}$$

$$\because \overline{ME} \perp \overline{AC}, MD = ME$$

$$\therefore AB = AC$$

$$\therefore \text{In } \triangle ABC: m(\angle ACB) = m(\angle ABC) = 65^\circ$$

$$\therefore m(\angle BAC) = 180^\circ - (65^\circ + 65^\circ) = 50^\circ$$

(The req.)

[b] $\because \overline{AB}, \overline{AC}$ are two tangents to the circle

$$\therefore AB = AC$$

$$\therefore \text{In } \triangle ABC:$$

$$m(\angle ABC) = m(\angle ACB) = \frac{180^\circ - 50^\circ}{2} = 65^\circ$$

 $\because BCDE$ is a cyclic quadrilateral

$$\therefore m(\angle EBC) + m(\angle D) = 180^\circ$$

$$\therefore m(\angle EBC) = 180^\circ - 115^\circ = 65^\circ$$

$$\therefore m(\angle ABC) = m(\angle EBC)$$

$$\therefore \overline{BC} \text{ bisects } \angle ABE \quad (\text{Q.E.D.})$$

4

[a] $\because AB = CD$ (properties of the rectangle)

$$\because CE = CD \quad \therefore AB = CE$$

$$\therefore m(\widehat{AB}) = m(\widehat{CE}) \text{ and adding } m(\widehat{BE})$$

to both sides

$$\therefore m(\widehat{AE}) = m(\widehat{BC})$$

$$\therefore AE = BC \quad (\text{Q.E.D.})$$

[b] $\because \overline{AD}$ is a tangent to the circle.

$$\therefore m(\angle DAB) \text{ (tangency)} \\ = m(\angle ACB) \text{ (inscribed)} \quad (1)$$

 $\because \overline{XY} \parallel \overline{BC}$, \overline{YC} is a transversal.

$$\therefore m(\angle AYX) = m(\angle ACB) \\ \text{(corresponding angles)} \quad (2)$$

From (1) and (2):

$$\therefore m(\angle DAB) = m(\angle AYX)$$

$$\therefore \overline{AD} \text{ is a tangent to the circle passing through} \\ \text{the vertices of } \triangle AXY \quad (\text{Q.E.D.})$$

5

$$[a] \because m(\angle D) = \frac{1}{2} m(\angle AMB)$$

(inscribed and central angles subtended by \widehat{AB})

$$\therefore m(\angle D) = \frac{1}{2} \times 140^\circ = 70^\circ \quad (\text{First req.})$$

 $\because \overline{AC} \parallel \overline{DB}$, \overline{AD} is transversal

$$\therefore m(\angle DAC) + m(\angle D) = 180^\circ$$

(two interior angles in the same side of the transversal)

$$\therefore m(\angle DAC) = 180^\circ - 70^\circ = 110^\circ \quad (\text{Second req.})$$

[b] In $\triangle ABD$: $\because AB = AD$

$$\therefore m(\angle BDA) = m(\angle ABD) = 30^\circ$$

$$\therefore m(\angle A) = 180^\circ - (30^\circ + 30^\circ) = 120^\circ$$

$$\because m(\angle DCE) = m(\angle A) = 120^\circ$$

$$\therefore ABCD \text{ is a cyclic quadrilateral.} \quad (\text{Q.E.D.})$$

23 New valley

1

$$1 \text{ b} \quad 2 \text{ d} \quad 3 \text{ d} \quad 4 \text{ c} \quad 5 \text{ a} \quad 6 \text{ b}$$

2

[a] $\because ABCD$ is cyclic quadrilateral.

$$\therefore m(\angle ADC) = m(\angle ABE) = 100^\circ$$

In $\triangle ACD$:

$$\therefore m(\angle ACD) = 180^\circ - (100^\circ + 40^\circ) = 40^\circ$$

$$\therefore m(\angle CAD) = m(\angle ACD)$$

$$\therefore m(\widehat{CD}) = m(\widehat{AD}) \quad (\text{Q.E.D.})$$

Geometry

[b] $\therefore X$ is the midpoint of \overline{AB}

$$\therefore \overline{MX} \perp \overline{AB}$$

$$\therefore m(\angle AXM) = 90^\circ$$

$\therefore Y$ is the midpoint of \overline{AC}

$$\therefore \overline{MY} \perp \overline{AC}$$

$$\therefore m(\angle AYM) = 90^\circ$$

From the quadrilateral $AXMY$:

$$m(\angle DMH) = 360^\circ - (90^\circ + 90^\circ + 70^\circ) = 110^\circ$$

(First req.)

$$\therefore AB = AC$$

$$\therefore MX = MY$$

$$\therefore MD = MH = r$$

$$\therefore XD = YH$$

(Second req.)

3

[a] $\therefore \overline{AD}$ is a tangent to the circle.

$$\therefore m(\angle DAB) \text{ (tangency)}$$

$$= m(\angle ACB) \text{ (inscribed)}$$

(1)

$\therefore \overline{XY} \parallel \overline{BC}$, \overline{YC} is a transversal.

$$\therefore m(\angle AYX) = m(\angle ACB)$$

(corresponding angles) (2)

From (1) and (2) :

$$\therefore m(\angle DAB) = m(\angle AYX)$$

$\therefore \overline{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

$$[b] \therefore m(\angle BCD) = \frac{1}{2} m(\angle BMD)$$

(inscribed and central angles subtended by \widehat{BD})

$$\therefore m(\angle BCD) = \frac{1}{2} \times 130^\circ = 65^\circ$$

$\therefore \overline{AB} \parallel \overline{CD}$, \overline{BC} is a transversal.

$$\therefore m(\angle ABC) = m(\angle BCD) = 65^\circ$$

(alternate angles) (1)

$\therefore \overline{AB}$, \overline{AC} are two tangent-segments

$$\therefore AB = AC$$

$$\therefore m(\angle ACB) = m(\angle ABC) = 65^\circ$$

(2)

From (1) and (2) :

$$\therefore m(\angle ACB) = m(\angle BCD) = 65^\circ$$

$$\therefore \overline{CB} \text{ bisects } \angle ACD$$

(First req.)

In $\triangle ABC$:

$$m(\angle A) = 180^\circ - (65^\circ + 65^\circ) = 50^\circ \text{ (Second req.)}$$

4

[a] $\therefore \overline{DE} \parallel \overline{BC}$

$$\therefore m(\widehat{DB}) = m(\widehat{EC})$$

adding $m(\widehat{BC})$ to both sides.

$$\therefore m(\widehat{DC}) = m(\widehat{EB})$$

$$\therefore m(\angle DAC) = m(\angle BAE)$$

(Q.E.D.)

[b] $\therefore m(\widehat{AX}) = m(\widehat{AY})$

$$\therefore m(\angle ACX) = m(\angle ABY)$$

and they are drawn on \overline{ED} and on one side of it.

$\therefore BCED$ is a cyclic quadrilateral.

(Q.E.D. 1)

$$\therefore m(\angle DEB) = m(\angle DCB)$$

$$\therefore m(\angle XCB) = m(\angle XAB)$$

(two inscribed angles subtended by \widehat{XB})

$$\therefore m(\angle DEB) = m(\angle XAB)$$

(Q.E.D. 2)

5

[a] State by yourself.

[b] $\therefore \overline{CD}$ is a diameter in the circle.

$$\therefore m(\angle CXD) = 90^\circ$$

$$\therefore \overline{CD} \perp \overline{AB}$$

$$\therefore m(\angle BEC) = 90^\circ$$

$$\therefore m(\angle CXD) = m(\angle BEC)$$

$\angle BEC$ is an exterior angle of the figure $XYEC$

$\therefore XYEC$ is a cyclic quadrilateral. (Q.E.D. 1)

$$\therefore m(\angle DYB) = m(\angle XCD)$$

(1)

$$\therefore m(\angle DBX) = m(\angle XCD)$$

(2)

(two inscribed angles subtended by \widehat{XD})

From (1) and (2) :

$$\therefore m(\angle DYB) = m(\angle DBX)$$

(Q.E.D. 2)

24 South Sinai

1

1 a

2 b

3 c

4 d

5 a

6 b

2

$$[a] \therefore m(\widehat{AB}) = 50^\circ$$

$$\therefore m(\angle D) = \frac{1}{2} m(\widehat{AB}) = \frac{1}{2} \times 50^\circ = 25^\circ$$

(First req.)

$$\therefore m(\angle AMB) = m(\widehat{AB}) = 50^\circ$$

(Second req.)

$$[b] \therefore m(\widehat{BC}) = m(\widehat{AD})$$

adding $m(\widehat{AC})$ to both sides

$$\therefore m(\widehat{AB}) = m(\widehat{CD}) \therefore AB = CD \text{ (Q.E.D.)}$$

Answers of Final Examinations

3

- [a] $\because r_1 = 5 \text{ cm.}, r_2 = 3 \text{ cm.}$
 $\therefore r_1 + r_2 = 5 + 3 = 8 \text{ cm.}$
 $\therefore r_1 + r_2 = MN$
 \therefore The two circles are touching externally.
- [b] $\because \overline{AB}$ is a tangent-segment to the circle.
 $\therefore \overline{AC}$ is a diameter of it.
 $\therefore \overline{AB} \perp \overline{AC}$
 $\therefore m(\angle BAC) = 90^\circ$ (1)
 $\because m(\angle ACD) = \frac{1}{2} m(\angle AMD)$
 (inscribed and central angles subtended by \widehat{AD})
 $\therefore m(\angle ACD) = \frac{1}{2} \times 60^\circ = 30^\circ$ (2)
 In $\triangle ABC$:
 $m(\angle ABC) = 180^\circ - (90^\circ + 30^\circ) = 60^\circ$ (First req.)
 From (1) and (2):
 $\therefore AB = \frac{1}{2} BC$ (Second req.)

4

- [a] In $\triangle ABC$: $\because m(\angle B) = m(\angle C)$
 $\therefore AB = AC$
 $\because D$ is midpoint of \overline{AB} $\therefore \overline{MD} \perp \overline{AB}$
 $\because E$ is midpoint of \overline{AC} $\therefore \overline{ME} \perp \overline{AC}$
 $\therefore MD = ME$ (Q.E.D.)
- [b] In $\triangle ABE$: $\because AB = AE$
 $\therefore m(\angle AEB) = m(\angle B)$
 $\because m(\angle D) = m(\angle B)$
 (properties of parallelogram)
 $\therefore m(\angle AEB) = m(\angle D)$
 \therefore The figure AECD is a cyclic quadrilateral.
 (Q.E.D.)

5

- [a] $\because \overline{AB}, \overline{AC}$ are two tangents to the circle.
 $\therefore AB = AC$
 \therefore In $\triangle ABC$:
 $m(\angle ABC) = m(\angle ACB) = \frac{180^\circ - 50^\circ}{2} = 65^\circ$
 $\because BCDE$ is a cyclic quadrilateral.
 $\therefore m(\angle EBC) + m(\angle D) = 180^\circ$
 $\therefore m(\angle EBC) = 180^\circ - 115^\circ = 65^\circ$

- $\therefore m(\angle ABC) = m(\angle EBC)$
 $\therefore \overline{BC}$ bisects $\angle ABE$ (Q.E.D. 1)
 $\because m(\angle BEC)$ (inscribed)
 $= m(\angle ABC)$ (tangency) $= 65^\circ$
 $\therefore m(\angle EBC) = m(\angle BEC)$
 \therefore In $\triangle BCE$: $CB = CE$ (Q.E.D. 2)
- [b] $\because m(\widehat{BC}) = 2m(\angle A) = 2 \times 30^\circ = 60^\circ$
 $\therefore m(\angle E) = \frac{1}{2} [m(\widehat{AD}) - m(\widehat{BC})]$
 $\therefore 50^\circ = \frac{1}{2} [m(\widehat{AD}) - 60^\circ]$
 $\therefore 100^\circ = m(\widehat{AD}) - 60^\circ$
 $\therefore m(\widehat{AD}) = 160^\circ$ (First req.)
 $\therefore m(\angle AFD) = \frac{1}{2} [m(\widehat{AD}) + m(\widehat{BC})]$
 $\therefore m(\angle AFD) = \frac{1}{2} [160^\circ + 60^\circ] = 110^\circ$ (Second req.)

25 : North Sinai

1

- 1 c 2 a 3 b 4 b 5 c 6 c

2

- [a] $\because AB = CD, \overline{MW} \perp \overline{AB}, \overline{MH} \perp \overline{CD}$
 $\therefore MX = MY$
 $\because MW = MH = r$
 $\therefore WX = HY$ (Q.E.D.)
- [b] $\because \overline{CD} \parallel \overline{BA}$ $\therefore m(\widehat{AC}) = m(\widehat{BC})$
 $\therefore AC = BC$ (First req.)
 $\because \overline{AB}$ is a diameter of the circle
 $\therefore m(\angle ACB) = 90^\circ$
 In $\triangle ABC$: $\therefore m(\angle B) = m(\angle A) = \frac{180^\circ - 90^\circ}{2} = 45^\circ$
 (Second req.)

3

- [a] State by yourself.
- [b] $\because D$ is the midpoint of \overline{BW}
 $\therefore \overline{MD} \perp \overline{BW}$
 $\therefore m(\angle WDM) = 90^\circ$
 $\because \overline{AC}$ is a tangent to the circle
 $\therefore \overline{AC} \perp \overline{BC}$ $\therefore m(\angle ACM) = 90^\circ$
 $\therefore m(\angle WDM) + m(\angle ACM) = 180^\circ$

Geometry

∴ The figure ADCM is a cyclic quadrilateral.

(Q.E.D. 1)

∵ $\angle CMH$ is an exterior angle of the cyclic quadrilateral ADCM

$$\therefore m(\angle CMH) = m(\angle A) \quad (1)$$

$$\therefore m(\angle CBH) = \frac{1}{2} m(\angle CMH) \quad (2)$$

(inscribed and central angles subtended by \widehat{BC})

From (1) and (2):

$$\therefore m(\angle CBH) = \frac{1}{2} m(\angle A) \quad (Q.E.D. 2)$$

4

$$[a] \therefore m(\angle A) = \frac{1}{2} [m(\widehat{CH}) - m(\widehat{BD})]$$

$$\therefore 30^\circ = \frac{1}{2} [80^\circ - m(\widehat{BD})]$$

$$\therefore 60^\circ = 80^\circ - m(\widehat{BD})$$

$$\therefore m(\widehat{BD}) = 80^\circ - 60^\circ = 20^\circ$$

∵ \widehat{BC} is a diameter in the circle

$$\therefore m(\widehat{BC}) = 180^\circ$$

$$\therefore m(\widehat{DH}) = 360^\circ - [180^\circ + 20^\circ + 80^\circ] = 80^\circ$$

(The req.)

$$[b] \therefore m(\angle BDC) \text{ (inscribed)} \\ = m(\angle ABC) \text{ (tangency)} = 70^\circ$$

∵ \widehat{AB} , \widehat{AC} are two tangents

$$\therefore AB = AC$$

$$\therefore m(\angle ABC) = m(\angle ACB) = 70^\circ$$

In $\triangle ABC$:

$$\therefore m(\angle BAC) = 180^\circ - (70^\circ + 70^\circ) = 40^\circ$$

(The req.)

5

$$[a] \text{ In } \triangle ABD: \therefore AB \approx AD$$

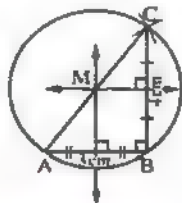
$$\therefore m(\angle ABD) \approx m(\angle ADB) = 30^\circ$$

$$\therefore m(\angle A) \approx 180^\circ - 2 \times 30^\circ = 120^\circ$$

$$\therefore m(\angle A) + m(\angle C) = 120^\circ + 60^\circ = 180^\circ$$

∴ ABCD is a cyclic quadrilateral. (Q.E.D.)

[b]



We can draw one circle only.

26 Red Sea

1

$$1 \text{ c} \quad 2 \text{ b} \quad 3 \text{ a} \quad 4 \text{ d} \quad 5 \text{ c} \quad 6 \text{ c}$$

2

$$[a] \therefore AB = CD, \overline{MX} \perp \overline{AB}, \overline{MY} \perp \overline{CD}$$

$$\therefore MX = MY$$

$$\therefore MH = MF = r \quad \therefore HX = FY \quad (Q.E.D.)$$

$$[b] \therefore m(\angle ADB) = \frac{1}{2} m(\widehat{AB}) = \frac{1}{2} \times 110^\circ = 55^\circ$$

∵ ABCD is a cyclic quadrilateral.

$$\therefore m(\angle HBC) = m(\angle CDB) + m(\angle ADB) \\ = 30^\circ + 55^\circ = 85^\circ \quad (\text{The req.})$$

3

$$[a] \text{ In } \triangle BMC: \therefore MB = MC = r$$

$$\therefore m(\angle MCB) = m(\angle MBC) = 25^\circ$$

$$\therefore m(\angle BMC) = 180^\circ - (25^\circ + 25^\circ) = 130^\circ$$

$$\therefore m(\angle BAC) = \frac{1}{2} m(\angle BMC)$$

(inscribed and central angles subtended by \widehat{BC})

$$\therefore m(\angle BAC) = \frac{1}{2} \times 130^\circ = 65^\circ \quad (\text{The req.})$$

$$[b] \text{ In } \triangle ABC: \therefore AB = AC$$

$$\therefore m(\angle ACB) = m(\angle ABC) = 50^\circ$$

$$\therefore m(\angle A) = 180^\circ - (50^\circ + 50^\circ) = 80^\circ$$

$$\therefore m(\angle A) + m(\angle D) = 80^\circ + 100^\circ = 180^\circ$$

∴ ABDC is a cyclic quadrilateral. (Q.E.D.)

4

$$[a] \therefore \overline{MN} \text{ is the line of centres}$$

∵ \overline{AB} is the common chord

$$\therefore \overline{AB} \perp \overline{MN} \quad \therefore m(\angle AXN) = 90^\circ$$

∵ The sum of the measures of the interior angles of the quadrilateral CDNX = 360°

$$\therefore m(\angle CDN) = 360^\circ - (125^\circ + 55^\circ + 90^\circ) = 90^\circ$$

$$\therefore \overline{ND} \perp \overline{CD}$$

∴ \overline{CD} is a tangent to the circle N at D (Q.E.D.)

$$[b] \therefore \overline{AX} \text{ is a common tangent for two circles}$$

$$\therefore m(\angle BDA) \text{ (inscribed)}$$

$$= m(\angle BAX) \text{ (tangency)}$$

Answers of Final Examinations

$\therefore m(\angle CHA)$ (inscribed)
 $= m(\angle CAX)$ (tangency)
 $\therefore m(\angle BDA) = m(\angle CHA)$
 and they are corresponding angles
 $\therefore \overline{BD} \parallel \overline{CH}$ (Q.E.D.)

5

[a] $\therefore m(\widehat{BD}) = 2m(\angle C)$
 $\therefore m(\widehat{BD}) = 2 \times 26^\circ = 52^\circ$
 $\therefore m(\angle A) = \frac{1}{2} [m(\widehat{CH}) - m(\widehat{BD})]$
 $\therefore 40^\circ = \frac{1}{2} [m(\widehat{CH}) - 52^\circ]$
 $\therefore 80^\circ = m(\widehat{CH}) - 52^\circ$
 $\therefore m(\widehat{CH}) = 80^\circ + 52^\circ = 132^\circ$ (First req.)
 $\therefore m(\angle HXC) = \frac{1}{2} [m(\widehat{CH}) + m(\widehat{BD})]$
 $= \frac{1}{2} [132^\circ + 52^\circ] = 92^\circ$ (Second req.)

[b] $\therefore \overline{AB}$, \overline{AC} are two tangents to the circle
 $\therefore AB = AC$
 $\therefore m(\angle ABC) = m(\angle ACB) = \frac{180^\circ - 70^\circ}{2} = 55^\circ$
 $\therefore m(\angle BHC)$ (inscribed)
 $= m(\angle ABC)$ (tangency) $= 55^\circ$
 $\therefore BCDH$ is a cyclic quadrilateral.
 $\therefore m(\angle CBH) + m(\angle CDH) = 180^\circ$
 $\therefore m(\angle CBH) = 180^\circ - 125^\circ = 55^\circ$
 In $\triangle BCH$: $\therefore m(\angle BHC) = m(\angle CBH)$
 $\therefore CB = CH$ (Q.E.D.)

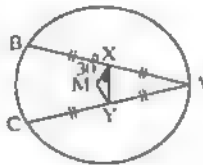
27 Matrouh

1

1 c 2 c 3 d 4 b 5 c 6 b

2

[a]



$\therefore X$ is the midpoint of \overline{AB}
 $\therefore \overline{MX} \perp \overline{AB}$
 $\therefore Y$ is the midpoint of \overline{AC}
 $\therefore \overline{MY} \perp \overline{AC}$

$\therefore AB = AC$ $\therefore MX = MY$
 $\therefore \triangle MXY$ is an isosceles triangle. (Q.E.D.)
 [b] $\therefore \overline{AF} \parallel \overline{DE}$, \overline{AB} is a transversal.
 $\therefore m(\angle BAF) = m(\angle AED)$
 (alternate angles) (1)
 $\therefore m(\angle C)$ (inscribed)
 $= m(\angle BAF)$ (tangency) (2)
 From (1) and (2) : $\therefore m(\angle C) = m(\angle AED)$
 $\therefore DEBC$ is a cyclic quadrilateral. (Q.E.D.)

3

[a] $\therefore m(\angle D) = \frac{1}{2} m(\angle M)$
 (inscribed and central angles subtended by \widehat{BC})
 $\therefore m(\angle D) = \frac{1}{2} \times 100^\circ = 50^\circ$
 $\therefore \angle ABD$ is an exterior angle of $\triangle BCD$
 $\therefore m(\angle ABD) = m(\angle BDC) + m(\angle DCB)$
 $\therefore m(\angle DCB) = 120^\circ - 50^\circ = 70^\circ$ (The req.)
 [b] $\therefore \overline{CA}$ and \overline{CB} are two tangents to the circle.
 $\therefore \overline{MA} \perp \overline{AC}$ $\therefore m(\angle MAC) = 90^\circ$
 $\therefore \overline{MB} \perp \overline{BC}$
 $\therefore m(\angle MBC) = 90^\circ$
 $\therefore m(\angle MAC) + m(\angle MBC) = 180^\circ$
 $\therefore ACBM$ is a cyclic quadrilateral.
 $\therefore \angle DMB$ is an exterior angle of it
 $\therefore m(\angle DMB) = m(\angle ACB)$ (Q.E.D.)

[a] $\therefore \overline{AD}$ is a tangent to the circle.

$\therefore m(\angle DAB)$ (tangency)
 $= m(\angle ACB)$ (inscribed) (1)

$\therefore \overline{XY} \parallel \overline{BC}$, \overline{YC} is a transversal.
 $\therefore m(\angle AYX) = m(\angle ACB)$
 (corresponding angles) (2)

From (1) and (2) : $\therefore m(\angle DAB) = m(\angle AYX)$
 $\therefore \overline{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

[b] $\therefore \overline{DE} \parallel \overline{BC}$

$\therefore m(\widehat{DB}) = m(\widehat{EC})$ adding $m(\widehat{BC})$ to both sides
 $\therefore m(\widehat{DC}) = m(\widehat{EB})$
 $\therefore m(\angle DAC) = m(\angle BAE)$ (Q.E.D.)

Geometry

5

[a] Prove by yourself.

[b] $\because \overline{AB}, \overline{AC}$ are two tangents to the circle

$$\therefore AB = AC$$

$$\therefore m(\angle ABC) = m(\angle ACB) = \frac{180^\circ - 70^\circ}{2} = 55^\circ$$

$$\begin{aligned} \therefore m(\angle CEB) \text{ (inscribed)} \\ = m(\angle CBA) \text{ (tangency)} = 55^\circ \end{aligned}$$

, $\because BCDE$ is a cyclic quadrilateral

$$\therefore m(\angle CBE) + m(\angle CDE) = 180^\circ$$

$$\therefore m(\angle CBE) = 180^\circ - 125^\circ = 55^\circ$$

$$\text{In } \triangle EBC : \therefore m(\angle CEB) = m(\angle CBE)$$

$$\therefore CB = CE \quad (\text{Q.E.D.1})$$

$$\therefore m(\angle ACB) = m(\angle CBE) = 55^\circ$$

and they are alternate angles

$$\therefore \overline{AC} \parallel \overline{BE} \quad (\text{Q.E.D.2})$$



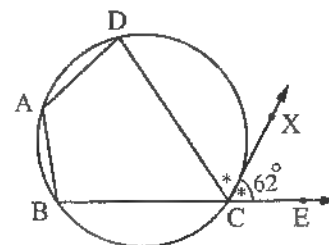
Answer the following questions :

1 Choose the correct answer :

- (1) The measure of the inscribed angle is the measure of the central angle , subtended by the same arc.
 (a) half (b) third (c) quarter (d) double
- (2) It is possible to draw a circle passing through the vertices of a
 (a) trapezium. (b) parallelogram. (c) rectangle. (d) rhombus.
- (3) The centre of the inscribed circle of any triangle is the point of intersection of its
 (a) altitudes. (b) medians.
 (c) axes of symmetry of its sides. (d) bisectors of its interior angles.
- (4) If the two circles M and N are touching internally , the radius length of one of them = 3 cm. and $MN = 8$ cm. , then the radius length of the other circle = cm.
 (a) 12 (b) 11 (c) 6 (d) 5

(5) In the opposite figure :

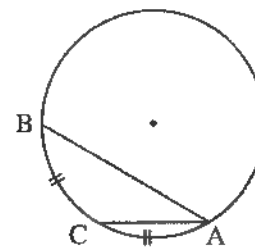
If $E \in \overrightarrow{BC}$, \overrightarrow{CX} bisects $\angle DCE$
 , $m(\angle XCE) = 62^\circ$
 , then $m(\angle A) = \dots\dots\dots$



- (a) 62° (b) 118° (c) 56° (d) 124°

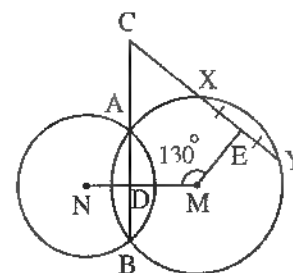
(6) In the opposite figure :

If C is the midpoint of \widehat{AB}
 , then $AB \dots\dots\dots 2 AC$
 (a) $<$ (b) $>$ (c) \geq (d) $=$



2 [a] In the opposite figure :

If E is the midpoint of \overline{XY}
 , $m(\angle EMN) = 130^\circ$
 , then find : $m(\angle C)$



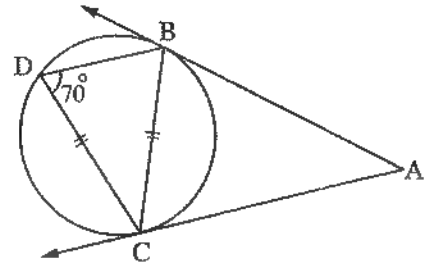
[b] In the opposite figure :

If \overrightarrow{AB} , \overrightarrow{AC} are two tangents to the circle at B, C

, $m(\angle D) = 70^\circ$, $CB = CD$

(1) Find : $m(\angle A)$

(2) Prove that : $\overline{BD} \parallel \overline{AC}$

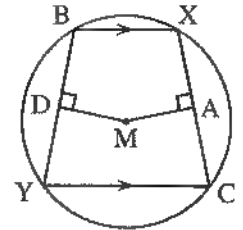


3 [a] In the opposite figure :

$\overline{XB} \parallel \overline{CY}$, $\overline{MA} \perp \overline{XC}$

, $\overline{MD} \perp \overline{BY}$

Prove that : $MA = MD$



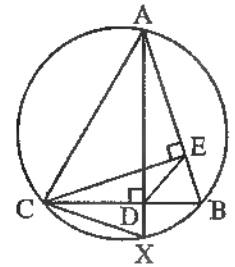
[b] In the opposite figure :

$\overline{CE} \perp \overline{AB}$, $\overline{AD} \perp \overline{BC}$ and intersects the circle at X

Prove that :

(1) AEDC is a cyclic quadrilateral.

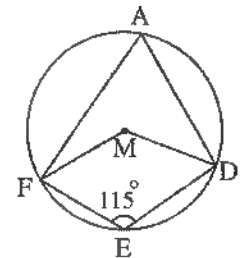
(2) \overline{CB} bisects $\angle ECX$



4 [a] In the opposite figure :

If $m(\angle DEF) = 115^\circ$

, then find : $m(\angle DMF)$

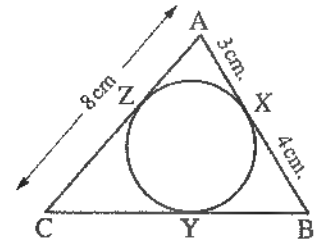


[b] In the opposite figure :

Inscribed circle of the triangle ABC touches its sides at X, Y and Z

If $AX = 3$ cm., $XB = 4$ cm., $AC = 8$ cm.

Find : The length of \overline{BC}

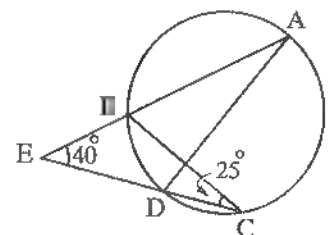


5 [a] In the opposite figure :

$\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$, $m(\angle C) = 25^\circ$

, $m(\angle E) = 40^\circ$

Find : $m(\angle ADC)$

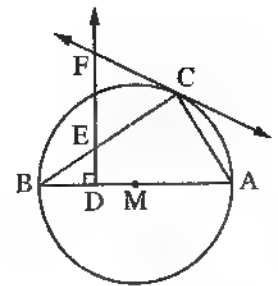


[b] In the opposite figure :

- \overline{AB} is a diameter in the circle M
- \overrightarrow{CF} is a tangent to the circle at C
- $\overrightarrow{DF} \perp \overline{AB}$ and intersects \overline{BC} at E

Prove that :

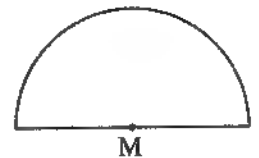
- (1) ADEC is a cyclic quadrilateral.
- (2) $\triangle FCE$ is an isosceles triangle.



Answer the following questions :

1 Choose the correct answer from those given :

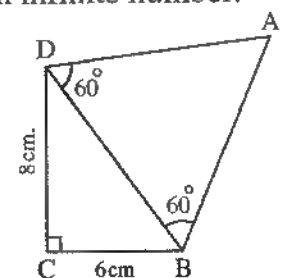
- (1) The two opposite angles in the cyclic quadrilateral are
 - (a) equal. (b) supplementary. (c) complementary. (d) alternate.
- (2) The opposite figure represents a semicircle its centre is M and its radius length is r length unit, then the area of the opposite figure = square units.
 - (a) $2\pi r$ (b) πr (c) πr^2 (d) $\frac{\pi r^2}{2}$
- (3) In a regular hexagon , the measure of the angle of its vertex equals
 - (a) 60° (b) 108° (c) 120° (d) 135°
- (4) If \overline{AB} is a line segment , then the number of circles can be drawn passing through A and B equals
 - (a) 1 (b) 2 (c) 3 (d) an infinite number.



(5) In the opposite figure :

The length of $\overline{AB} =$ cm.

- (a) $10\sqrt{3}$ (b) 10
- (c) 5 (d) $5\sqrt{3}$



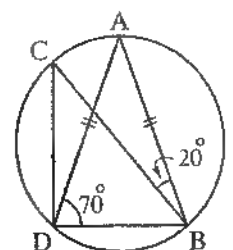
- (6) The inscribed angle which is opposite to the minor arc in a circle is
 - (a) acute. (b) right. (c) obtuse. (d) reflex.

2 [a] In the opposite figure :

$AB = AD$

- , $m(\angle ABC) = 20^\circ$
- , $m(\angle ADB) = 70^\circ$

Find : $m(\angle C)$, $m(\angle BDC)$



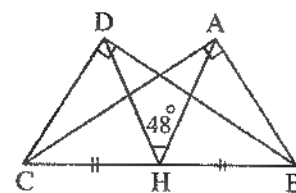
[b] In the opposite figure :

$$m(\angle BAC) = m(\angle BDC) = 90^\circ$$

, H is the midpoint of \overline{BC} and $m(\angle AHD) = 48^\circ$

(1) Prove that : ABCD is a cyclic quadrilateral.

(2) Find : $m(\angle ABD)$



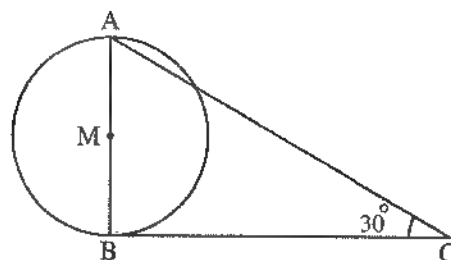
3 [a] In the opposite figure :

A circle M of circumference 44 cm.

, \overline{AB} is a diameter , \overline{BC} is a tangent at B

and $m(\angle ACB) = 30^\circ$

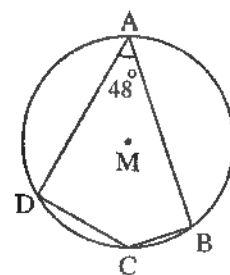
Find : The length of \overline{BC} ($\pi = \frac{22}{7}$)



[b] In the opposite figure :

If M is a circle , $m(\angle A) = 48^\circ$

Find : $m(\widehat{BD})$ the major

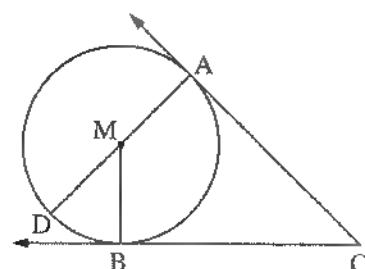


4 [a] In the opposite figure :

\overline{AD} is a diameter in a circle M

, \overline{CA} and \overline{CB} are two tangents to the circle M ,
touch it at A and B respectively.

Prove that : $m(\angle DMB) = m(\angle ACB)$



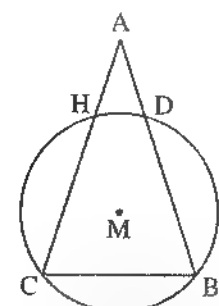
[b] In the opposite figure :

ABC is a triangle in which $AB = AC$

, \overline{BC} is a chord in the circle M

, if \overline{AB} and \overline{AC} cut the circle at D and H respectively.

Prove that : $m(\widehat{DB}) = m(\widehat{HC})$

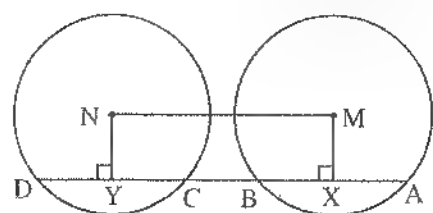


5 [a] In the opposite figure :

M and N are two congruent circles

, $AB = CD$

Prove that : The figure MXYN is a rectangle.



[b] ABCD is a quadrilateral inscribed in a circle , H is a point outside the circle and \overrightarrow{HA} and \overrightarrow{HB} are two tangents to the circle at A and B , if $m(\angle AHB) = 70^\circ$ and $m(\angle ADC) = 125^\circ$, prove that :

(1) $AB = AC$

(2) \overrightarrow{AC} is a tangent to the circle passing through the points A , B and H

3 El-Kalyoubia Governorate



Answer the following questions :

1 Choose the correct answer :

(1) If the area of the circle is $9\pi \text{ cm}^2$, then its radius length = cm.

- (a) 9 (b) 2 (c) (-3) (d) 3

(2) The number of symmetric axes of a square =

- (a) 1 (b) 2 (c) 3 (d) 4

(3) If M is a circle of a diameter length equals 14 cm. , $MA = (2x + 3) \text{ cm}$. where A lies on the circle , then $x = \dots\dots\dots$

- (a) 5 (b) 3 (c) 2 (d) 1

(4) The ratio between the measure of the inscribed angle and the measure of the central angle subtended by the same arc =

- (a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) 1 : 3

(5) If ABCD is a cyclic quadrilateral and $m(\angle B) = \frac{1}{2} m(\angle D)$, then $m(\angle B) = \dots\dots\dots$

- (a) 90° (b) 60° (c) 120° (d) 180°

(6) If the figure ABCD ~ the figure XYZL , then $m(\angle B) = m(\angle \dots\dots\dots)$

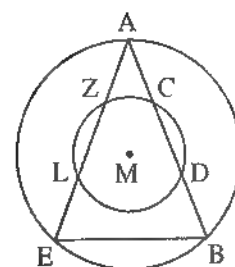
- (a) X (b) Y (c) Z (d) L

2 [a] In the opposite figure :

Two concentric circles at M

, $m(\angle ABE) = m(\angle AEB)$

Prove that : $CD = ZL$

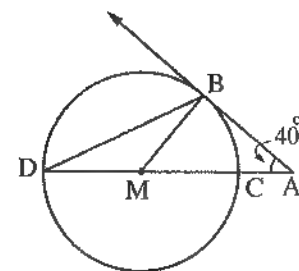


[b] In the opposite figure :

\overrightarrow{AB} is a tangent to the circle M

, $m(\angle A) = 40^\circ$

Find with proof : $m(\angle BDC)$



- 3 [a] Using your geometric tools , draw \overline{AB} with a length of 4 cm. , then draw a circle passing through the two points A and B whose radius length is 3 cm.

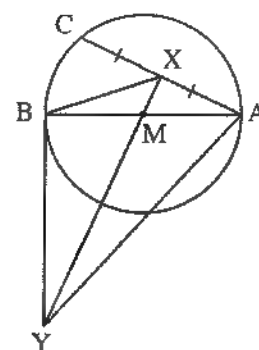
What are the possible solutions ? (Don't remove the arcs)

[b] In the opposite figure :

\overline{AB} is a diameter in the circle M

, X is the midpoint of \overline{AC} and \overline{XM} intersecting the tangent of the circle at B in Y

Prove that : The figure AXBY is a cyclic quadrilateral.

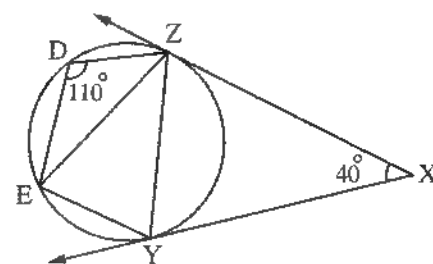


4 [a] In the opposite figure :

\overrightarrow{XY} and \overrightarrow{XZ} are two tangents to the circle at the two points Y and Z , $m(\angle X) = 40^\circ$

, $m(\angle D) = 110^\circ$

Prove that : $m(\angle ZYE) = m(\angle ZEY)$



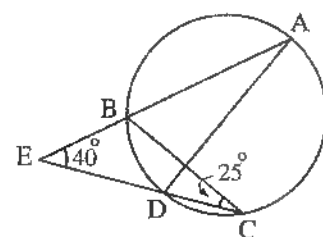
[b] In the opposite figure :

$m(\angle E) = 40^\circ$, $m(\angle C) = 25^\circ$

Find with proof :

(1) $m(\angle ADC)$

(2) $m(\widehat{AC})$

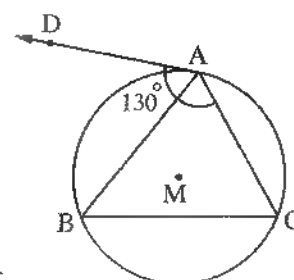


5 [a] In the opposite figure :

\overrightarrow{AD} is the tangent to the circle M at A

, $m(\angle DAC) = 130^\circ$

Find with proof : $m(\angle B)$



[b] ABCD is a quadrilateral drawn in a circle , $E \in \overrightarrow{AB}$, $E \notin \overline{AB}$

, $m(\widehat{AB}) = 110^\circ$, $m(\angle CBE) = 85^\circ$

Find with proof : $m(\angle BDC)$



Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

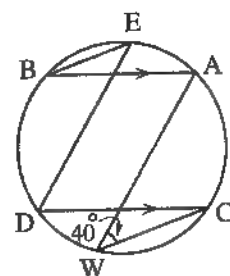
- (1) The two tangents which are drawn from the two endpoints of a diameter of a circle are
 (a) parallel. (b) perpendicular. (c) coincide. (d) intersecting.
- (2) The number of the axes of symmetry of the semicircle the number of the axes of symmetry of the isosceles triangle.
 (a) > (b) < (c) = (d) \geq

(3) In the opposite figure :

$\overline{AB} \parallel \overline{CD}$, $m(\angle AWC) = 40^\circ$,

then $m(\angle DEB) = \dots\dots\dots$

- (a) 50° (b) 40°
 (c) 30° (d) 45°



- (4) A circle, its radius length $(2x + 6)$ cm. and the straight line L is at distance $(x + 2)$ cm. from its centre where $x > 0$, then L is

- (a) outside the circle. (b) a tangent to the circle.
 (c) a secant to the circle. (d) passing through the centre.

- (5) If the straight line $\overleftrightarrow{AB} \cap$ the circle $M = \{A, B\}$

, then $\overleftrightarrow{AB} \cap$ the surface of the circle $M = \dots\dots\dots$

- (a) $\{A, B\}$ (b) \overline{AB} (c) \overleftrightarrow{AB} (d) \overleftrightarrow{BA}

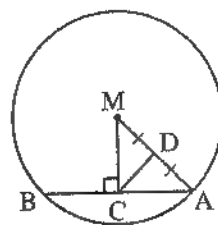
(6) In the opposite figure :

$CD = 3$ cm., $\overline{MC} \perp \overline{AB}$

, D is the midpoint of \overline{MA}

then the area of the circle M = π cm².

- (a) 3 (b) 6 (c) 9 (d) 36



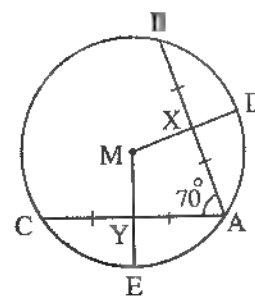
2 [a] In the opposite figure :

\overline{AB} and \overline{AC} are two chords equal in length at the circle M

, X is the midpoint of \overline{AB}

, Y is the midpoint of \overline{AC} , $m(\angle A) = 70^\circ$

- (1) Find : $m(\angle DME)$ (2) Prove that : $XD = YE$

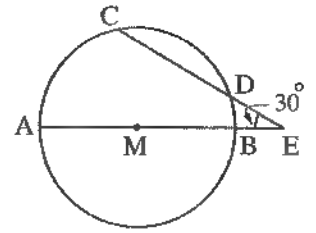


[b] In the opposite figure :

\overline{AB} is a diameter in the circle M

, $\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$, $m(\angle E) = 30^\circ$, $m(\widehat{AC}) = 80^\circ$

Find : $m(\widehat{CD})$

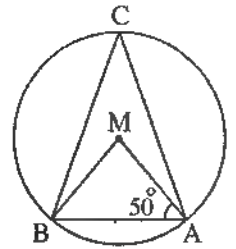


- 3 [a] Complete : The measure of the inscribed angle equals the measure of the central angle by the same arc.

[b] In the opposite figure :

M is a circle , $m(\angle MAB) = 50^\circ$

Find : $m(\angle C)$

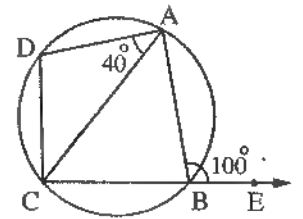


- 4 [a] In the opposite figure :

$m(\angle ABE) = 100^\circ$

, $m(\angle CAD) = 40^\circ$

Prove that : $\triangle DAC$ is an isosceles triangle.

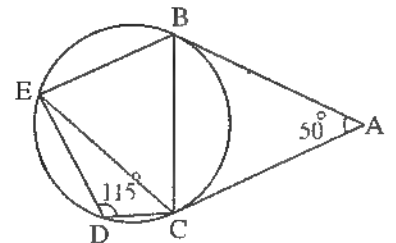


[b] In the opposite figure :

\overline{AB} and \overline{AC} are two tangent-segments to the circle at B and C

, $m(\angle A) = 50^\circ$, $m(\angle D) = 115^\circ$

Prove that : (1) \overrightarrow{BC} bisects $\angle ABE$ (2) $CB = CE$



- 5 [a] Complete : The measure of the inscribed angle in a semicircle equals $^\circ$

[b] In the opposite figure :

ABC and DCE are two equilateral triangles

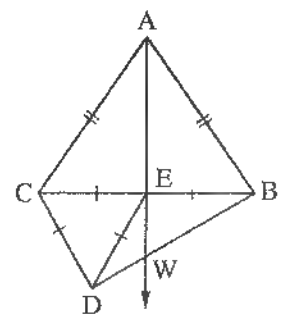
, E is the midpoint of \overline{BC} , $\overrightarrow{AE} \cap \overrightarrow{BD} = \{W\}$

(1) Prove that : \overline{AC} is a tangent-segment to the circle

which passes through the vertices of $\triangle CED$

(2) Prove that : CDWE is a cyclic quadrilateral.

(3) Find : The centre of the circle which passes through the vertices of the quadrilateral CDWE





Answer the following questions : (Calculator is allowed)

1 Choose the correct answer :

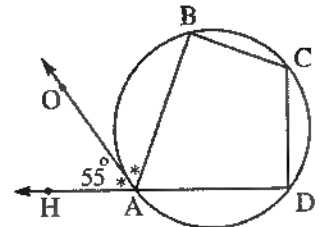
(1) In the opposite figure :

$H \in \overrightarrow{DA}$, \overrightarrow{AO} bisects $\angle HAB$

, $m(\angle HAO) = 55^\circ$

, then $m(\angle C) = \dots\dots\dots$

- (a) 55° (b) 75° (c) 110° (d) 125°



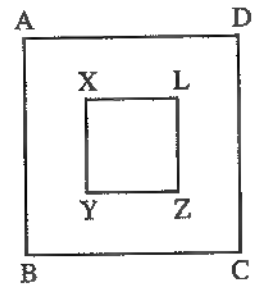
(2) In the opposite figure :

If the side length of the square $ABCD = 7$ cm.

and the side length of the square $XYZL = 3$ cm.

, then the area of the shaded part = $\dots\dots\dots$ cm^2

- (a) $(7 - 3)$ (b) $4(7 - 3)$
(c) $(7 - 3)^2$ (d) $(7^2 - 3^2)$



(3) If $\overrightarrow{AB} \cap$ the circle $M = \{A, B\}$, then $\overrightarrow{AB} \cap$ the surface of the circle $M = \dots\dots\dots$

- (a) \overrightarrow{AB} (b) \overline{AB} (c) $\{A, B\}$ (d) \overline{AB}

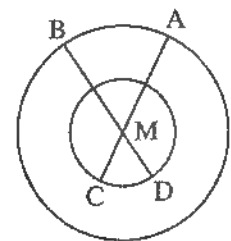
(4) In the opposite figure :

Two concentric circles with centre M

, the radii lengths of them are 6 cm. and 3 cm.

, if $m(\widehat{AB}) = 60^\circ$, then $m(\widehat{DC}) = \dots\dots\dots$

- (a) 60° (b) 30° (c) 120° (d) 40°



(5) If \overline{MA} and \overline{MB} are two perpendicular radii in a circle M and the area of triangle

$AMB = 8 \text{ cm}^2$, then the length of radius of this circle = $\dots\dots\dots$

- (a) 8 cm. (b) 16 cm. (c) 4 cm. (d) 2 cm.

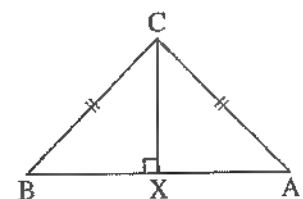
(6) In the opposite figure :

$CA = CB$, $\overline{CX} \perp \overline{AB}$

, $AB = 2 CX$

, then $m(\angle A) = \dots\dots\dots$

- (a) 30° (b) 60° (c) 90° (d) 45°



2 [a] In the opposite figure :

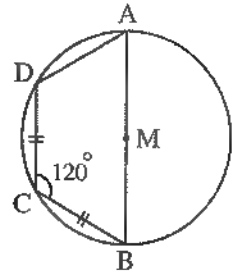
ABCD is a quadrilateral inscribed in the circle M

, $M \in \overline{AB}$, $CB = CD$

, $m(\angle BCD) = 120^\circ$

Find : (1) $m(\angle A)$

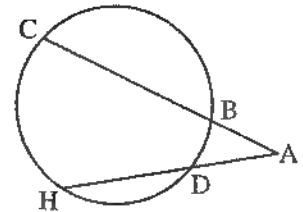
(2) $m(\angle D)$



[b] In the opposite figure :

If $m(\widehat{HC}) = 100^\circ$, $m(\widehat{BD}) = 30^\circ$

Find : $m(\angle A)$



3 [a] In the opposite figure :

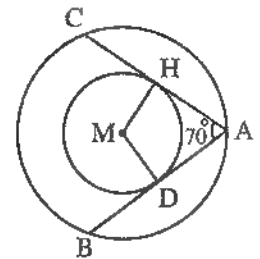
Two concentric circles at M

, \overline{AB} and \overline{AC} are two tangents to the smaller circle

, $m(\angle A) = 70^\circ$

(1) **Find :** $m(\angle DMH)$

(2) **Prove that :** $AB = AC$

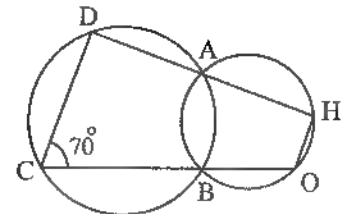


[b] In the opposite figure :

Two intersecting circles at A and B , $m(\angle C) = 70^\circ$

(1) **Find :** $m(\angle O)$

(2) **Prove that :** $\overline{CD} \parallel \overline{HO}$



4 [a] \overline{AB} is a diameter in the circle M , \overline{AC} is a chord such that $m(\angle BAC) = 30^\circ$

, draw \overline{BC} and draw $\overline{MD} \perp \overline{AC}$ and cut it at D

(1) **Prove that :** $\overline{MD} \parallel \overline{BC}$

(2) **Prove that :** The length \overline{BC} = the length of the radius of this circle.

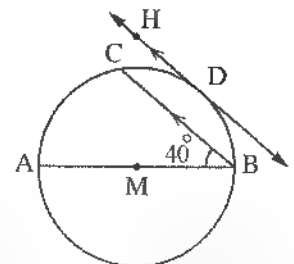
[b] In the opposite figure :

\overline{AB} is a diameter in the circle M

, $m(\angle B) = 40^\circ$, \overleftrightarrow{DH} is a tangent at D

, $\overleftrightarrow{DH} \parallel \overline{BC}$

Find : $m(\widehat{DC})$



5 [a] If circle with radius length 5 cm. , A is a point in its plane where $MA = (2X - 3)$ cm.

Find the value of X if A is located outside the circle.

[b] In the opposite figure :

\overline{AB} is a diameter of the circle M , H is a midpoint of a chord \overline{AC}

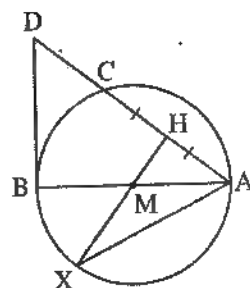
, \overline{BD} is a tangent to the circle at B

, \overline{HM} cuts the circle at X , **porve that :**

(1) MHDB is a cyclic quadrilateral.

(2) $m(\angle BAX) = \frac{1}{2} m(\angle D)$

(3) \overline{AB} is a tangent to the circle passing through the points B , C and D



6

El-Gharbia Governorate



Answer the following questions :

1 Choose the correct answer from those given :

(1) If the length of a diameter of a circle is 8 cm. and the straight line L at a distance of 4 cm. from its centre , then L is

(a) a secant to the circle at two points.

(b) lying outside the circle.

(c) a tangent to the circle.

(d) an axis of symmetry to the circle.

(2) In the opposite figure :

If M is the centre of the circle

, $m(\angle BMD) = 110^\circ$

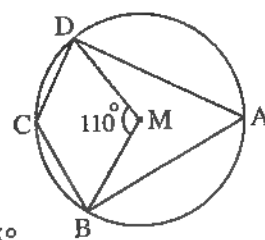
, then $m(\angle C) = \dots\dots\dots$

(a) 70°

(b) 110°

(c) 125°

(d) 55°



(3) In the opposite figure :

\overline{AB} is a tangent of the circle M

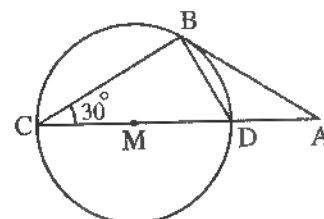
, then $m(\angle ABC) = \dots\dots\dots$

(a) 120°

(b) 110°

(c) 90°

(d) 30°



(4) The centre of the inscribed circle of any triangle is the intersection point

(a) its medians.

(b) its heights.

(c) the symmetric axes of its sides.

(d) bisectors of its interior angles.

(5) In the opposite figure :

$m(\widehat{AC}) = 50^\circ$, $\overline{AB} \parallel \overline{CD}$

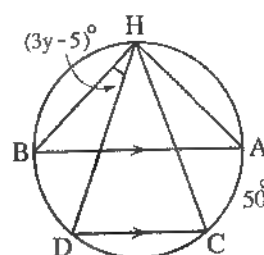
, then the value of $y = \dots\dots\dots$

(a) 5°

(b) 10°

(c) 15°

(d) 20°



(6) In the opposite figure :

$$MX = MY, m(\angle B) = 50^\circ$$

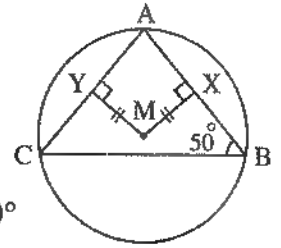
, then $m(\angle A) = \dots\dots\dots$

(a) 50°

(b) 60°

(c) 70°

(d) 80°

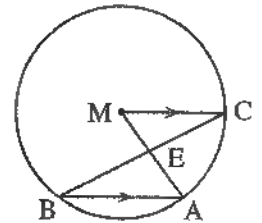


2 [a] In the opposite figure :

\overline{AB} is a chord in the circle M

, $\overline{CM} \parallel \overline{AB}$, $\overline{BC} \cap \overline{AM} = \{E\}$

Prove that : $BE > AE$



[b] In the opposite figure :

\overline{AB} and \overline{BC} are two chords in the circle M

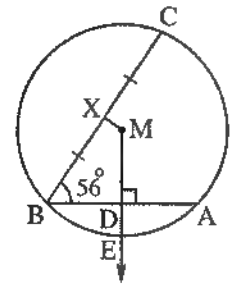
, its radius of length 5 cm. , $\overline{MD} \perp \overline{AB}$ and cuts \overline{AB}

at D and cuts the circle at E , X is midpoint of \overline{BC}

, $AB = 8$ cm. and $m(\angle ABC) = 56^\circ$

Find : (1) $m(\angle DMX)$

(2) The length of \overline{DE}

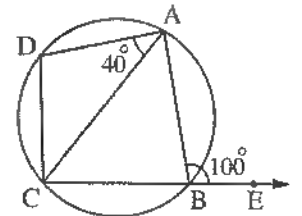


3 [a] In the opposite figure :

$$m(\angle ABE) = 100^\circ$$

$$, m(\angle CAD) = 40^\circ$$

Prove that : $m(\widehat{CD}) = m(\widehat{AD})$



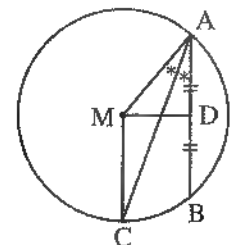
[b] In the opposite figure :

\overline{AB} is a chord in the circle M

, \overline{AC} bisects $\angle BAM$ and cuts the circle M at C

, D is midpoint of \overline{AB}

Prove that : $\overline{DM} \perp \overline{CM}$



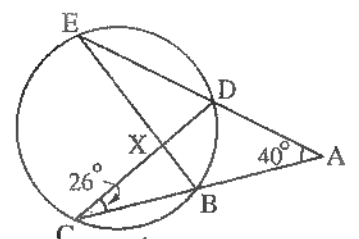
4 [a] In the opposite figure :

$$\overline{CB} \cap \overline{ED} = \{A\}, m(\angle A) = 40^\circ$$

$$, \overline{DC} \cap \overline{BE} = \{X\}, m(\angle BCD) = 26^\circ$$

Find : (1) $m(\widehat{CE})$

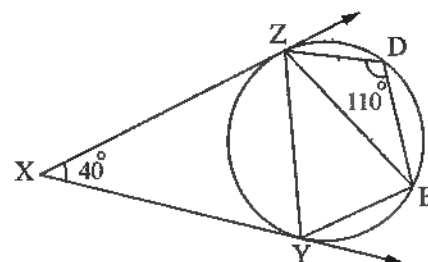
(2) $m(\angle EXC)$



[b] In the opposite figure :

\overrightarrow{XY} and \overrightarrow{XZ} are two tangents to the circle from the point X , $m(\angle X) = 40^\circ$
 , $m(\angle D) = 110^\circ$

Prove that : $m(\widehat{ZDE}) = m(\widehat{ZY})$

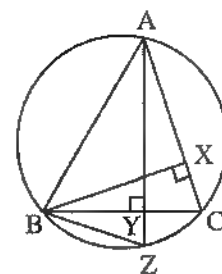


5 [a] In the opposite figure :

ABC is a triangle drawn in a circle
 , $\overrightarrow{BX} \perp \overrightarrow{AC}$, $\overrightarrow{AY} \perp \overrightarrow{BC}$ cuts it at Y and cuts the circle at Z

Prove that :

- (1) ABYX is a cyclic quadrilateral.
- (2) \overrightarrow{BC} bisects $\angle XBZ$



[b] In the opposite figure :

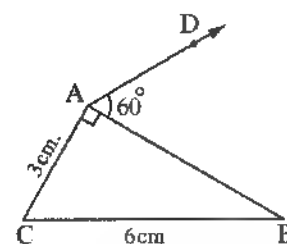
ABC is a right-angled triangle at A

, $AC = 3$ cm. , $BC = 6$ cm.

, $m(\angle BAD) = 60^\circ$

Prove that :

\overrightarrow{AD} is a tangent to the circle passing through the vertices of the triangle ABC



7 El-Dakahlia Governorate



Answer the following questions : (Calculator is allowed)

1 [a] Choose the correct answer from the given answers :

(1) M and N are two circles of radii lengths 9 cm. , 4 cm. , $MN = 5$ cm.
 , then the two circles are

- | | |
|--------------------------|--------------------------|
| (a) intersecting. | (b) touching internally. |
| (c) touching externally. | (d) distant. |

(2) The centres of all circles passing through the points A and B lie on

- | | |
|--|---|
| (a) \overline{AB} | (b) midpoint of \overline{AB} |
| (c) the symmetry axis of \overline{AB} | (d) the perpendicular to \overline{AB} from B |

(3) The measure of the inscribed angle which is drawn in a semicircle equals

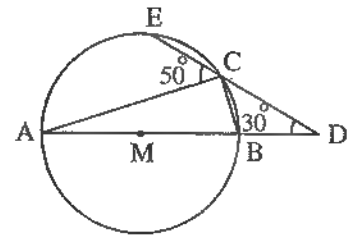
- | | | | |
|-----------------|----------------|----------------|-----------------|
| (a) 180° | (b) 90° | (c) 45° | (d) 100° |
|-----------------|----------------|----------------|-----------------|

[b] In the opposite figure :

\overline{AB} is a diameter in the circle M

, $m(\angle D) = 30^\circ$, $m(\angle ACE) = 50^\circ$

Find by proof : $m(\angle CBA)$



2 [a] Choose the correct answer from the given answers :

(1) In the opposite figure :

\overline{CB} and \overline{CD} are two tangent-segments at B and D

, $m(\angle C) = 70^\circ$

, then $m(\widehat{DB} \text{ the minor}) = \dots\dots\dots$

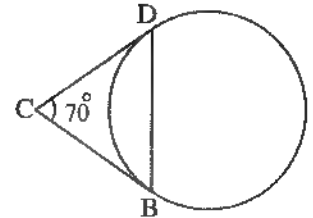
- (a) 180° (b) 90° (c) 100° (d) 110°

(2) \overline{AB} and \overline{CD} are two equal chords in length in the circle M, X and Y are the two midpoints of \overline{AB} and \overline{CD} respectively, $MX = 3$ cm., then $MY = \dots\dots\dots$ cm.

- (a) 3 (b) 6 (c) $\frac{3}{2}$ (d) 4

(3) The length of the arc which represents $\frac{1}{4}$ of the circle equals $\dots\dots\dots$

- (a) $4\pi r$ (b) $2\pi r$ (c) πr (d) $\frac{1}{2}\pi r$



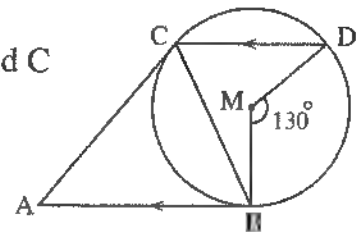
[b] In the opposite figure :

\overline{AB} and \overline{AC} are two tangent-segments to the circle M at B and C

, $\overline{AB} \parallel \overline{CD}$, $m(\angle BMD) = 130^\circ$

(1) Prove that : \overline{CB} bisects $\angle ACD$

(2) Find by proof : $m(\angle A)$



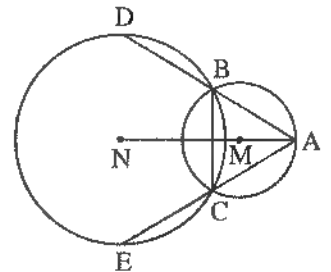
3 [a] Using the geometric tools, draw \overline{AB} with length 6 cm., then draw \overline{AC} where $m(\angle CAB) = 60^\circ$, draw the circle that passes through the points A, B and its centre lies on \overline{AC} and calculate the length of its radius (Don't remove the arcs).

[b] In the opposite figure :

M and N are two intersecting circles at B and C

, $A \in \overline{MN}$

Prove that : $BD = CE$



4 [a] In the opposite figure :

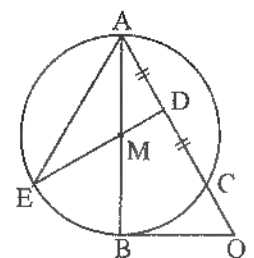
\overline{OB} is a tangent-segment to the circle M at B

, \overline{AB} is a diameter, D is the midpoint of \overline{AC}

Prove that :

(1) DOBM is a cyclic quadrilateral.

(2) $m(\angle AOB) = 2m(\angle BAE)$



(6) The sum of measures of the interior angles of the quadrilateral =

- (a) 90° (b) 180° (c) 270° (d) 360°

2 [a] In the opposite figure :

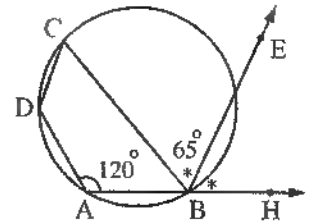
ABCD is a cyclic quadrilateral in which

$m(\angle A) = 120^\circ$, \overrightarrow{BE} bisects $\angle HBC$

, $m(\angle EBC) = 65^\circ$

Find with proof : (1) $m(\angle C)$

(2) $m(\angle D)$

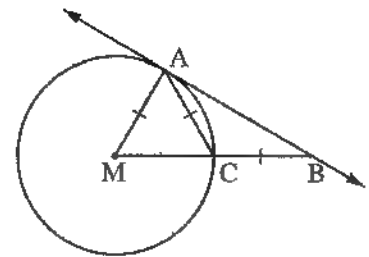


[b] In the opposite figure :

M is a circle, $AM = AC = BC$

Prove that :

\overrightarrow{AB} is a tangent to the circle at A

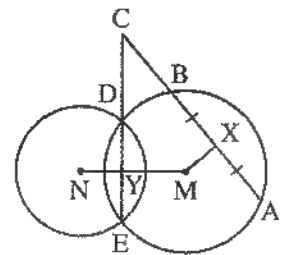


3 [a] In opposite figure :

X is the midpoint of \overline{AB} , $\overline{MN} \cap \overline{EC} = \{Y\}$

(1) **Prove that :** CXMY is a cyclic quadrilateral.

(2) **Find :** The centre of the circle which passes through the vertices of the figure CXMY



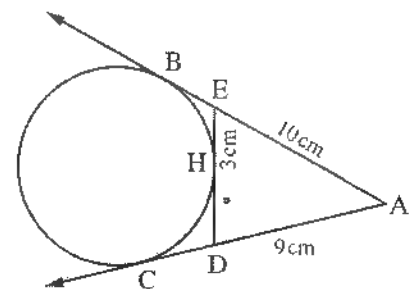
[b] In the opposite figure :

\overrightarrow{AB} , \overrightarrow{AC} are two tangents to a circle

, \overrightarrow{ED} is a tangent to the circle at H such that $AE = 10$ cm.

, $EH = 3$ cm. , $AD = 9$ cm.

Find : The length of \overline{ED}

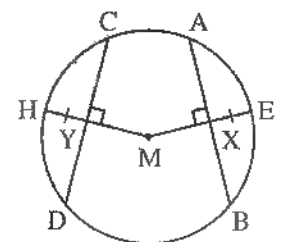


4 [a] In the opposite figure :

$\overline{ME} \perp \overline{AB}$, $\overline{MH} \perp \overline{CD}$

, $EX = YH$

Prove that : $AB = CD$



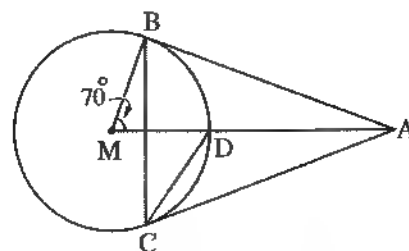
[b] Using geometric tools. Draw \overline{AB} its length is 6 cm. , then draw a circle passing through the two points A , B and its radius length is 3 cm.

How many circles can be drawn ?

5 [a] In the opposite figure :

\overline{AB} and \overline{AC} are two tangent-segments drawn from A
 $m(\angle AMB) = 70^\circ$

Find : (1) $m(\angle ABC)$ (2) $m(\angle ACD)$



[b] \overline{AB} and \overline{CD} are two equal chords in length in a circle

$\overline{AB} \cap \overline{CD} = \{E\}$, $m(\widehat{AD}) = 50^\circ$

(1) **Prove that :** $m(\widehat{AD}) = m(\widehat{BC})$ (2) **Find :** $m(\angle AED)$

9 Suez Governorate



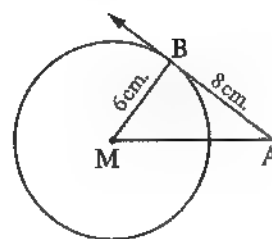
Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

(1) In the opposite figure :

\overline{AB} is a tangent to the circle M
 $MB = 6$ cm. , $AB = 8$ cm.
 , then $AM = \dots\dots\dots$ cm.

- (a) 5 (b) 10 (c) 12 (d) 13



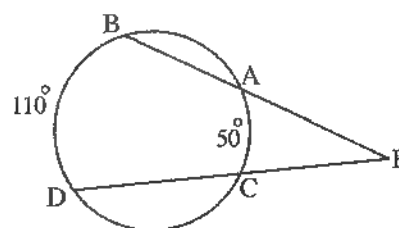
(2) If the two circles M and N are touching externally, the radius length of one of them is 5 cm. ,
 and $MN = 9$ cm. , then the radius length of the other circle equals $\dots\dots\dots$ cm.

- (a) 4 (b) 5 (c) 9 (d) 14

(3) In the opposite figure :

If $m(\widehat{AC}) = 50^\circ$, $m(\widehat{BD}) = 110^\circ$
 , then $m(\angle E) = \dots\dots\dots^\circ$

- (a) 60 (b) 50
 (c) 40 (d) 30



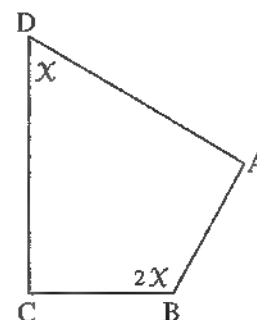
(4) A circle can be drawn passing the vertices of a $\dots\dots\dots$

- (a) rhombus. (b) rectangle. (c) trapezoid. (d) parallelogram.

(5) In the opposite figure :

ABCD is a cyclic quadrilateral , $m(\angle D) = x^\circ$, $m(\angle B) = 2x^\circ$
 , then $x = \dots\dots\dots$

- (a) 120° (b) 100°
 (c) 60° (d) 50°



(6) In the opposite figure :

In a circle M , $\overline{AB} \parallel \overline{CD}$

, $m(\angle BMD) = 80^\circ$

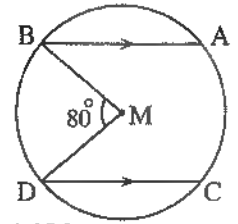
, then $m(\widehat{AC}) = \dots\dots\dots$

(a) 20°

(b) 40°

(c) 80°

(d) 160°

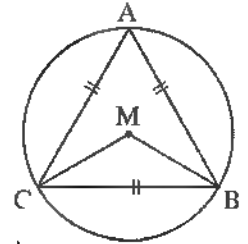


2 [a] In the opposite figure :

ABC is an equilateral triangle drawn inside a circle M

Find : (1) $m(\angle BAC)$

(2) $m(\angle BMC)$



[b] In the opposite figure :

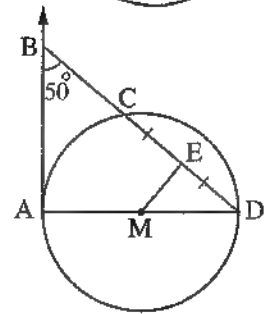
\overline{AD} is a diameter of the circle M

, \overline{AB} is a tangent touches it at A

, $m(\angle ABC) = 50^\circ$

, E is the midpoint of \overline{DC}

Find with proof : $m(\angle AME)$

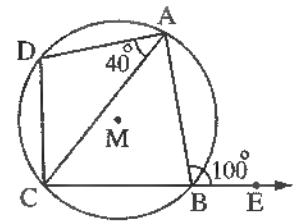


3 [a] In the opposite figure :

$m(\angle ABE) = 100^\circ$

, $m(\angle CAD) = 40^\circ$

Prove that : ADC is an isosceles triangle.



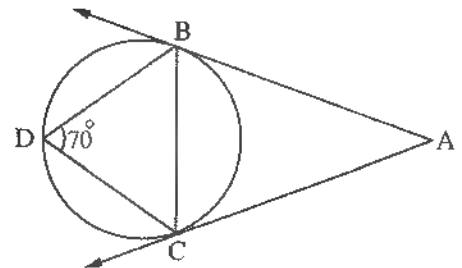
[b] In the opposite figure :

\overline{AB} , \overline{AC} are two tangents to the circle at B , C

, $m(\angle D) = 70^\circ$

Find : (1) $m(\angle ABC)$

(2) $m(\angle A)$

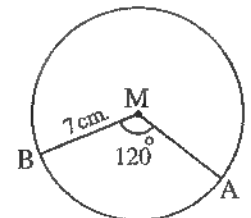


4 [a] In the opposite figure :

M is a circle with radius length 7 cm.

, $m(\angle AMB) = 120^\circ$

Find : The length of (\widehat{AB}) ($\pi = \frac{22}{7}$)



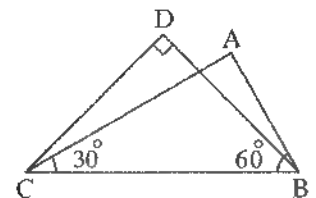
[b] In the opposite figure :

$m(\angle BDC) = 90^\circ$, $m(\angle ACB) = 30^\circ$

, $m(\angle ABC) = 60^\circ$

Prove that :

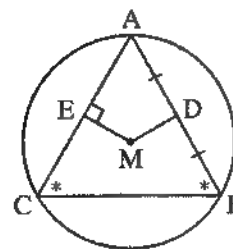
The points A , B , C and D have one circle passing through them.



5 [a] In the opposite figure :

Triangle ABC is inscribed in the circle M , in which
 $m(\angle B) = m(\angle C)$, D is the midpoint of \overline{AB}
 , $\overline{ME} \perp \overline{AC}$

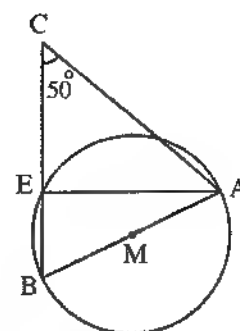
Prove that : $MD = ME$



[b] In the opposite figure :

\overline{AB} is a diameter of the circle M
 , $m(\angle C) = 50^\circ$

Find with proof : $m(\angle CAE)$



10 Port Said Governorate



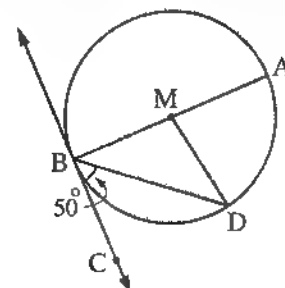
Answer the following questions :

1 Choose the correct answer from those given :

(1) In the opposite figure :

If $m(\angle CBD) = 50^\circ$
 , then $m(\angle AMD) = \dots\dots\dots$

- (a) 40° (b) 50°
 (c) 80° (d) 100°



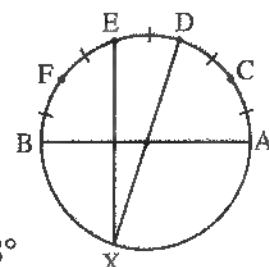
(2) A circle with diameter length $(2X + 5)$ cm. , the straight line L is distant from its centre by $(X + 2)$ cm. where $X > 0$, then the straight line is

- (a) a secant to the circle at two points. (b) lying outside the circle.
 (c) a tangent to the circle. (d) an axis of symmetry to the circle.

(3) In the opposite figure :

If \overline{AB} is a diameter in circle
 , $m(\widehat{AC}) = m(\widehat{CD}) = m(\widehat{DE}) = m(\widehat{EF}) = m(\widehat{FB})$
 , then $m(\angle DXE) = \dots\dots\dots$

- (a) 72° (b) 54° (c) 36° (d) 18°



(4) M and N are two intersecting circles their radii lengths are 5 cm. , 2 cm. , then $MN \in \dots\dots\dots$

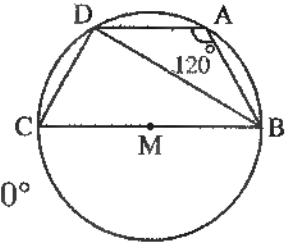
- (a) $[3, 7[$ (b) $]3, 7[$ (c) $]3, 7]$ (d) $[3, 7]$

(5) In the opposite figure :

If $m(\angle BAD) = 120^\circ$

, then $m(\angle CBD) = \dots\dots\dots$

- (a) 15° (b) 30° (c) 45° (d) 60°



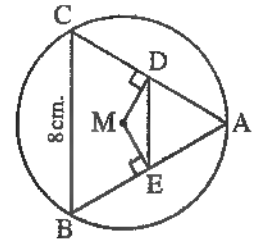
(6) The number of all common tangents drawn to two distant circles equals

- (a) 4 (b) 3 (c) 2 (d) 1

2 [a] Using the given data in the opposite figure :

(1) Prove that : $\overline{DE} \parallel \overline{CB}$

(2) Find : DE



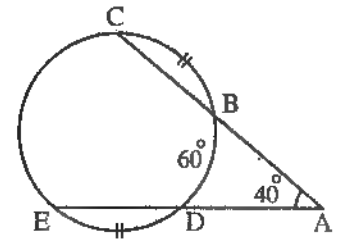
[b] In the opposite figure :

$m(\angle A) = 40^\circ$, $m(\widehat{BD}) = 60^\circ$

and $m(\widehat{BC}) = m(\widehat{DE})$

Find with proof :

$m(\widehat{EC})$ and $m(\widehat{BC})$



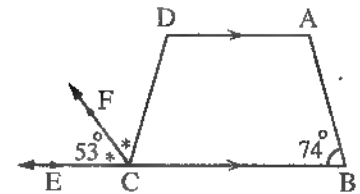
3 [a] Using the given data in the opposite figure :

Prove that :

ABCD is a cyclic quadrilateral.

[b] ABCD a parallelogram in which $AC = BC$

Prove that : \overrightarrow{CD} is a tangent to the circumcircle of the triangle ABC

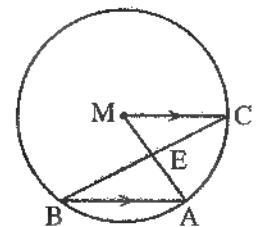


4 [a] In the opposite figure :

\overline{AB} is a chord in the circle M

, $\overline{CM} \parallel \overline{AB}$, $\overline{BC} \cap \overline{AM} = \{E\}$

Prove that : $BE > AE$



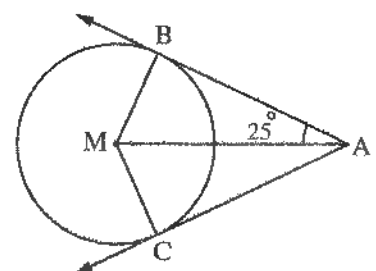
[b] In the opposite figure :

\overrightarrow{AB} and \overrightarrow{AC} are two tangents to the circle M

touch it at B and C respectively and $m(\angle BAM) = 25^\circ$

(1) Prove that : \overrightarrow{MA} bisects $(\angle BMC)$

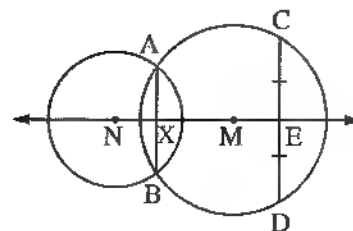
(2) Find : $m(\angle BMC)$.



5 [a] In the opposite figure :

The two circles M and N intersect at A and B
 \overline{CD} is a chord in the circle M cuts \overleftrightarrow{MN} at E
 , if E is the midpoint of \overline{CD}

Prove that : $\overline{AB} \parallel \overline{CD}$



[b] ABCD is a square , \overline{AX} bisects $\angle BAC$ and intersects \overline{BD} at X
 and \overline{DY} bisects $\angle CDB$ and intersects \overline{AC} at Y

Prove that : AXYD is a cyclic quadrilateral.



11 Damietta Governorate

Answer the following questions : (Calculator is allowed)

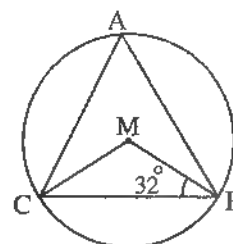
1 Choose the correct answer from the given ones :

- (1) ABC is a triangle having one symmetric line and its side lengths are 10 , 5 and X cm. , then X = cm.
 (a) 5 (b) 8 (c) 10 (d) 12
- (2) If the two circles M , N are touching internally , the length of one radius of them is 3 cm. , $MN = 8$ cm. , then the length of the radius of the other circle is cm.
 (a) 5 (b) 11 (c) 6 (d) 12
- (3) If the ratio between the measures of the angles of a triangle is 2 : 3 : 4 , then the measure of the greatest angle is
 (a) 40° (b) 90° (c) 45° (d) 80°

(4) In the opposite figure :

M is a circle , $m(\angle MBC) = 32^\circ$
 , then $m(\widehat{BC} \text{ the minor}) = \dots\dots\dots$

- (a) 116° (b) 23° (c) 58° (d) 64°



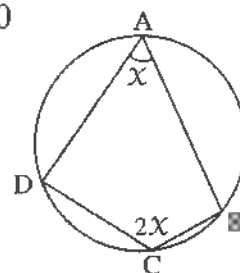
(5) A rectangular picture its length is 60 cm. and its width is 40 cm. We need to make a wooden frame its width is 5 cm. , then its total area is cm^2

- (a) 3050 (b) 3500 (c) 2925 (d) 3250

(6) In the opposite figure :

$m(\angle A) = X^\circ$, $m(\angle C) = 2X^\circ$
 , then $X = \dots\dots\dots$

- (a) 60° (b) 50° (c) 80° (d) 20°



- 2** [a] A, B are two points where $AB = 6$ cm., draw a circle of radius length 5 cm. and passes through the two points A, B

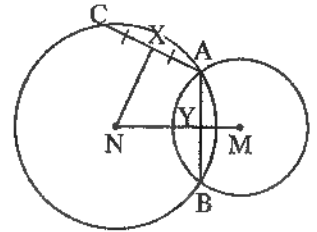
Find : (1) The number of circles can be drawn.

(2) The distance from the centre to \overline{AB} by proof.

[b] In the opposite figure :

M, N are two intersecting circles at A, B, $\overleftrightarrow{MN} \cap \overline{AB} = \{Y\}$
 $AB = AC$, if X is the midpoint of \overline{AC}

Prove that : $NX = NY$



- 3** [a] \overline{AB} , \overline{AC} are two chords in a circle

If X and Y are the two midpoints of \widehat{AB} , \widehat{AC} respectively, \overline{XY} cuts \overline{AB} at D, \overline{AC} at H

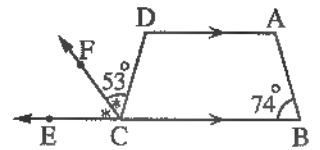
Prove that : $AD = AH$

[b] In the opposite figure :

$\overline{AD} \parallel \overline{BC}$, $m(\angle B) = 74^\circ$, $m(\angle DCF) = 53^\circ$

\overrightarrow{CF} bisects $\angle DCE$

Prove that : ABCD is a cyclic quadrilateral.

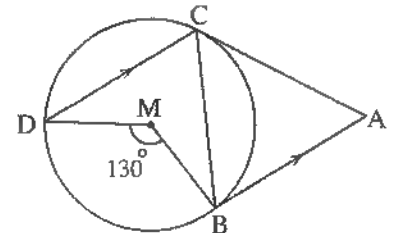


- 4** [a] **In the opposite figure :**

\overline{AB} and \overline{AC} are two tangent-segments to the circle M

$\overline{AB} \parallel \overline{CD}$, $m(\angle BMD) = 130^\circ$

Prove that : \overrightarrow{CB} bisects $\angle ACD$



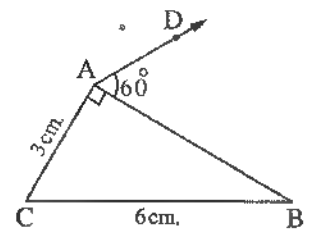
[b] In the opposite figure :

$m(\angle BAC) = 90^\circ$, $m(\angle DAB) = 60^\circ$

$AC = 3$ cm., $BC = 6$ cm.

Prove that :

\overline{AD} is a tangent to the circle passing through the vertices of the triangle ABC

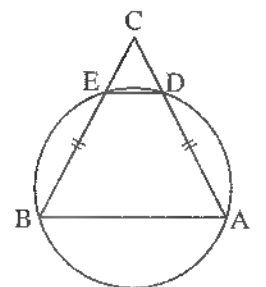


- 5** [a] **In the opposite figure :**

\overline{AD} and \overline{BE} are two equal chords in length in the circle

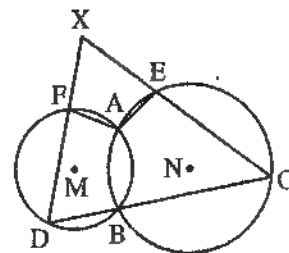
$\overrightarrow{AD} \cap \overrightarrow{BE} = \{C\}$

Prove that : $CD = CE$

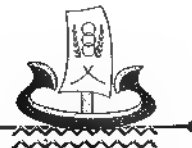


[b] In the opposite figure :

Two intersecting circles at A and B
 \overline{CD} passes through the point B and intersects
 the two circles at C and D
Prove that : AFXE is a cyclic quadrilateral.



12 Kafr El-Sheikh Governorate



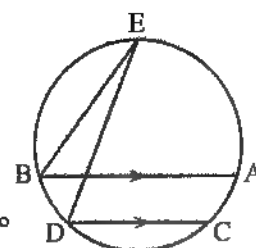
Answer the following questions : (Calculator is allowed)

1 [a] Choose the correct answer from those given :

(1) In the opposite figure :

If $m(\widehat{AC}) = 30^\circ$, $\overline{AB} \parallel \overline{CD}$
 , then $m(\angle BED) = \dots\dots\dots$

- (a) 10° (b) 15° (c) 30° (d) 60°



(2) The two tangents drawn from the two ends of a diameter of a circle are

- (a) parallel. (b) equal in length. (c) congruent. (d) intersecting.

(3) M and N are two intersecting circles their radii lengths are 5 cm. , 2 cm.

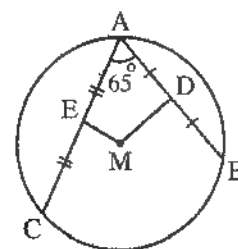
, then $MN \in \dots\dots\dots$

- (a) $]3, 7[$ (b) $[3, 7[$ (c) $]3, 7]$ (d) $[3, 7]$

[b] In the opposite figure :

\overline{AB} , \overline{AC} are two chords in the circle M ,
 D , E are the two midpoints of \overline{AB} , \overline{AC} respectively
 and $m(\angle BAC) = 65^\circ$

Find : $m(\angle DME)$

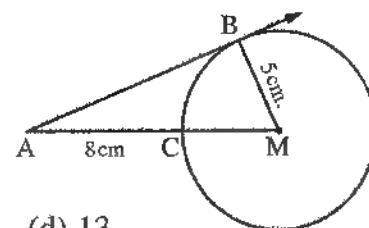


2 [a] Choose the correct answer from those given :

(1) In the opposite figure :

\overrightarrow{AB} is a tangent to the circle M
 , if $MB = 5$ cm. , $AC = 8$ cm. , then $AB = \dots\dots\dots$ cm.

- (a) 5 (b) 10 (c) 12 (d) 13



(2) The centre of the circumcircle of any triangle is the point of intersection of

- (a) the interior bisectors of its angles. (b) the exterior bisectors of its angles.
 (c) its heights. (d) the symmetric axes of its sides.

(3) The measure of the arc which represents $\frac{1}{3}$ the measure of the circle equals

- (a) 60° (b) 90° (c) 120° (d) 240°

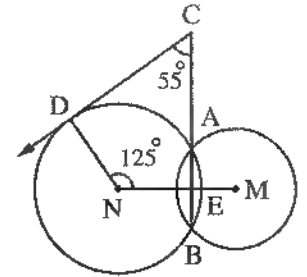
[b] In the opposite figure :

M and N are two intersecting circles at A and B

, $C \in \overrightarrow{BA}$, $D \in$ the circle N

, $m(\angle MND) = 125^\circ$ and $m(\angle BCD) = 55^\circ$

Prove that : \overrightarrow{CD} is a tangent to the circle N at D



3 [a] State three cases of the cyclic quadrilateral.

[b] ABCD is a quadrilateral in which $AB = AD$, $m(\angle ABD) = 30^\circ$ and $m(\angle C) = 60^\circ$

Prove that : ABCD is a cyclic quadrilateral.

4 [a] **Prove that :** The two tangent-segments drawn to a circle from a point outside it are equal in length.

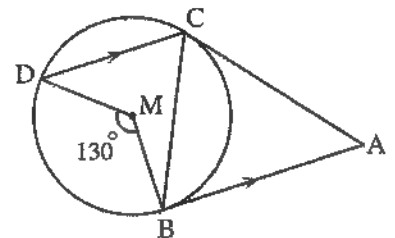
[b] In the opposite figure :

\overline{AB} and \overline{AC} are two tangent-segments to the circle M

, $\overline{AB} \parallel \overline{CD}$, $m(\angle BMD) = 130^\circ$

(1) **Prove that :** \overrightarrow{CB} bisects $\angle ACD$

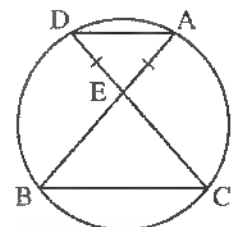
(2) **Find :** $m(\angle A)$ with proof.



5 [a] In the opposite figure :

$\overline{AB} \cap \overline{CD} = \{E\}$, $EA = ED$

Prove that : $EB = EC$



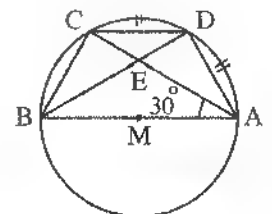
[b] In the opposite figure :

\overline{AB} is a diameter of a circle M , $C \in$ the circle

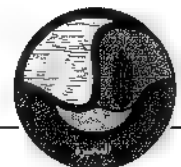
, $m(\angle CAB) = 30^\circ$, D is the midpoint of \widehat{AC} , $\overline{DB} \cap \overline{AC} = \{E\}$

(1) **Find :** $m(\angle BDC)$, $m(\angle ABD)$ with proof.

(2) **Prove that :** $\triangle ABE$ is an isosceles triangle.



13 El-Beheira Governorate



Answer the following questions : (Calculators are permitted)

1 Choose the correct answer from those given :

(1) The distance between the two points $(6, 0)$, $(-4, 0)$ equals length units.

(a) 10

(b) 10

(c) 2

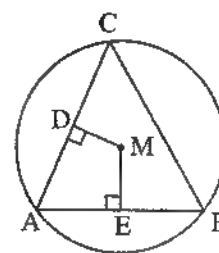
(d) 24

- (2) If the length of a diameter of a circle is 7 cm. , and the straight line L at a distance of 3.5 cm. from its centre , then L is
- (a) a secant to the circle at two points. (b) lying outside the circle.
(c) a tangent to the circle. (d) an axis of symmetry to the circle.
- (3) If \overline{AB} is a diameter of a circle , where A (3 , - 5) , B (5 , 1) , then the centre of the circle is
- (a) (4 , - 2) (b) (4 , 2) (c) (2 , 2) (d) (8 , - 2)
- (4) The inscribed angle which is opposite to the minor arc in a circle is ..
- (a) reflex. (b) right. (c) obtuse. (d) acute.
- (5) It is possible to draw a circle passing through the vertices of a
- (a) trapezium. (b) rhombus. (c) parallelogram. (d) rectangle.
- (6) The number of tangents can be drawn from a point lies on a circle equals
- (a) one. (b) two. (c) four. (d) infinite number.

2 [a] In the opposite figure :

ABC is a triangle drawn inside a circle of centre M
 $\overline{MD} \perp \overline{AC}$, $\overline{ME} \perp \overline{AB}$
 $\overline{BC} = 8$ cm.

- (1) Prove that : $\overline{DE} \parallel \overline{CB}$ (2) Find : DE

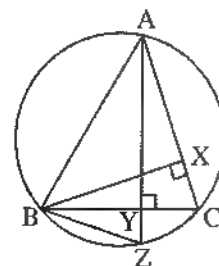


[b] In the opposite figure :

ABC is a triangle drawn inside a circle , $\overline{BX} \perp \overline{AC}$
 $\overline{AY} \perp \overline{BC}$ cuts it at Y and cuts the circle at Z

Prove that :

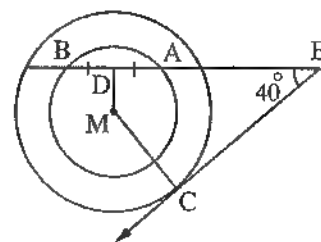
- (1) ABYX is a cyclic quadrilateral.
 (2) \overline{BC} bisects $\angle XBZ$



3 [a] In the opposite figure :

Two concentric circles of centre M
 \overline{EC} is a tangent to the greater circle
 \overline{EB} cuts the smaller circle at A , B
 D is the midpoint of \overline{AB} and $m(\angle CED) = 40^\circ$

Find with proof : $m(\angle DMC)$



- [b] \overline{AB} , \overline{CD} are two parallel chords in a circle M , E is the midpoint of \overline{AB} ,
 \overline{EM} is drawn to cut \overline{CD} at F **Prove that :** $FC = FD$

4 [a] In the opposite figure :

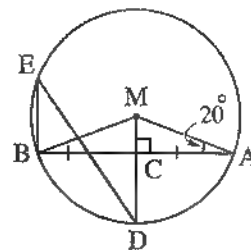
$$\overrightarrow{MC} \cap \overrightarrow{AB} = \{C\}, \overrightarrow{MC} \perp \overrightarrow{AB}$$

, \overrightarrow{MC} intersects the circle at D

$$, m(\angle MAB) = 20^\circ$$

Find : (1) $m(\widehat{AD})$

(2) $m(\angle DEB)$



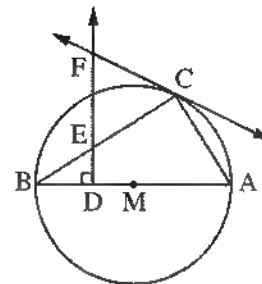
[b] In the opposite figure :

\overline{AB} is a diameter of a circle M

, \overrightarrow{CF} is a tangent of the circle at C and $\overrightarrow{DE} \perp \overrightarrow{AB}$

Prove that : (1) ADEC is a cyclic quadrilateral.

(2) $FE = FC$



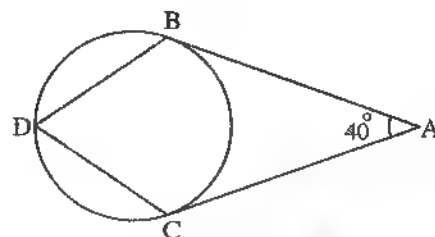
5 [a] Find the measure of the arc which represents $\frac{1}{3}$ its circle , then calculate the length of this arc if the length of the radius is 7 cm. ($\pi = \frac{22}{7}$)

[b] In the opposite figure :

\overline{AB} , \overline{AC} are two tangents to the circle at B , C

$$\text{and } m(\angle A) = 40^\circ$$

Find with proof : $m(\angle D)$



14 El-Fayoum Governorate



Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

(1) If the straight line L is a tangent to the circle of diameter 8 cm. , then the distance between L and the centre equals cm.

- (a) 3 (b) 4 (c) 6 (d) 8

(2) The angle whose measure is 50° complements an angle of measure

- (a) 90° (b) 130° (c) 50° (d) 40°

(3) The inscribed angle which is opposite to the minor arc in a circle is

- (a) reflex. (b) obtuse. (c) right. (d) acute.

(4) ABC is a triangle in which $AB = AC$, $m(\angle C) = 40^\circ$, then $m(\angle A) = \dots\dots\dots$

- (a) 40° (b) 80° (c) 100° (d) 120°

(5) The number of the symmetry axes of square is

- (a) 1 (b) 2 (c) 3 (d) 4

(6) In the opposite figure :

In the circle M , if $m(\angle AMC) = 140^\circ$

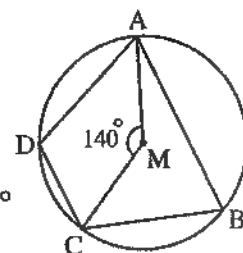
, then $m(\angle ADC) = \dots\dots\dots$

(a) 40°

(b) 70°

(c) 110°

(d) 140°



2 [a] In the opposite figure :

Triangle ABC is inscribed in circle M , in which :

$m(\angle B) = m(\angle C)$, X is the midpoint of \overline{AB}

, $\overline{MY} \perp \overline{AC}$

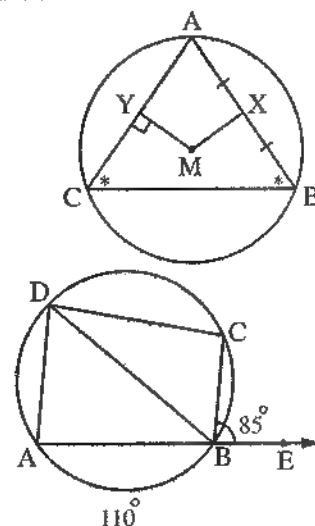
Prove that : $MX = MY$

[b] In the opposite figure :

$E \in \overrightarrow{AB}$, $E \notin \overline{AB}$, $m(\widehat{AB}) = 110^\circ$

, $m(\angle CBE) = 85^\circ$

Find : $m(\angle BDC)$



3 [a] In the opposite figure :

\overline{AC} is a diameter in a circle M , $B \in$ the circle M

, $m(\angle BAC) = 40^\circ$

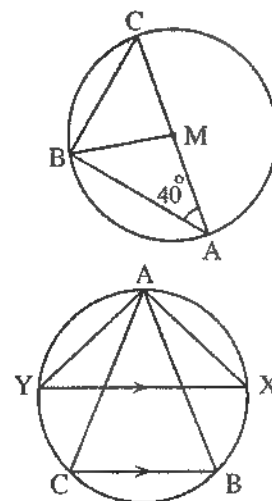
Find : $m(\angle CBM)$

[b] In the opposite figure :

ABC is an inscribed triangle inside a circle

, $\overline{XY} \parallel \overline{BC}$

Prove that : $m(\angle XAC) = m(\angle BAY)$.



4 [a] In the opposite figure :

M and N are two intersecting circles at A and B , $C \in \overrightarrow{BA}$

, $D \in$ the circle N and $m(\angle MND) = 125^\circ$

, $m(\angle BCD) = 55^\circ$

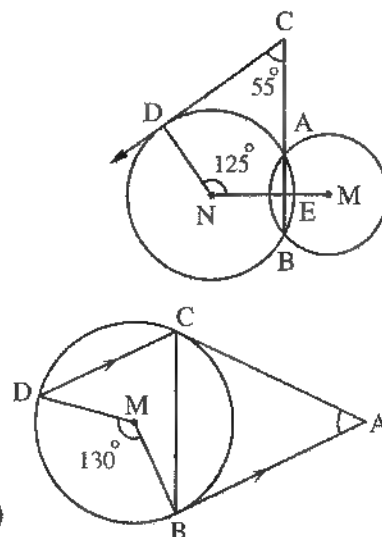
Prove that : \overleftrightarrow{CD} is a tangent to circle N at D

[b] In the opposite figure :

\overline{AB} and \overline{AC} are two tangent-segments to the circle M

, $\overline{AB} \parallel \overline{CD}$, $m(\angle BMD) = 130^\circ$

(1) Prove that : \overleftrightarrow{CB} bisects $\angle ACD$ (2) Find : $m(\angle A)$

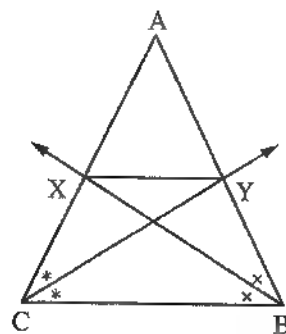


5 [a] In the opposite figure :

ABC is a triangle in which $AB = AC$
 \overrightarrow{BX} bisects $\angle B$ and intersect \overline{AC} at X
 \overrightarrow{CY} bisects $\angle C$ and intersect \overline{AB} at Y

Prove that : BCXY is a cyclic quadrilateral

and prove that : $\overline{XY} \parallel \overline{BC}$



[b] ABC is a triangle inscribed in a circle, \overleftrightarrow{AD} is a tangent to the circle at A
 $X \in \overline{AB}$, $Y \in \overline{AC}$ where $\overline{XY} \parallel \overline{BC}$ **Prove that :** \overleftrightarrow{AD} is a tangent to the circle passing through the points A, X and Y

15 Beni Suef Governorate



Answer the following questions : (Calculator is allowed)

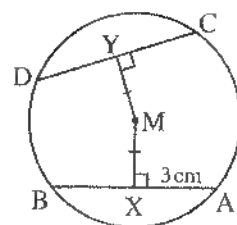
1 Choose the correct answer from those given :

- (1) It is impossible to draw a circle passing through the vertices of
 (a) a triangle. (b) a square. (c) a rhombus. (d) a rectangle.
- (2) If m_1 and m_2 are the slopes of two perpendicular straight lines , then .
 (a) $m_1 + m_2 = 0$ (b) $m_1 - m_2 = -1$ (c) $m_1 = m_2$ (d) $m_1 \times m_2 = -1$
- (3) M and N are two circles touching internally , their radii lengths are 3 cm. , and 5 cm.
 , then MN = cm.
 (a) 2 (b) 3 (c) 5 (d) 8
- (4) The point of concurrence of the medians of the triangle divides each median in the ratio from its base.
 (a) 2 : 1 (b) 1 : 2 (c) 2 : 3 (d) 1 : 3
- (5) The measure of the arc which represents $\frac{1}{3}$ the measure of the circle equals .
 (a) 60° (b) 90° (c) 120° (d) 240°
- (6) The area of the rhombus whose diagonal lengths are 8 cm. and 10 cm.
 equals cm^2 .
 (a) 2 (b) 18 (c) 40 (d) 80

2 [a] In the opposite figure :

$\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$, $MX = MY$
 and $AX = 3 \text{ cm}$.

Find : The length of \overline{CD}



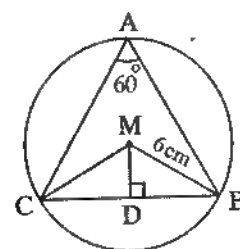
- [b] Two concentric circles M , \overline{AB} is a chord in the larger circle and intersects the smaller circle at C, D , draw $\overline{ME} \perp \overline{AB}$ Prove that : $AC = BD$

3 [a] In the opposite figure :

In the circle M , $m(\angle A) = 60^\circ$

, $\overline{MD} \perp \overline{BC}$, $MB = 6$ cm.

Find with proof : The length of \overline{MD}

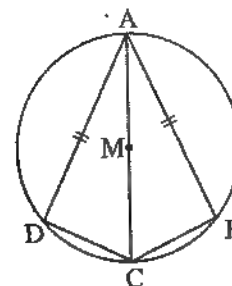


[b] In the opposite figure :

\overline{AC} is a diameter in the circle M

, $AB = AD$

Prove that : $m(\widehat{BC}) = m(\widehat{CD})$

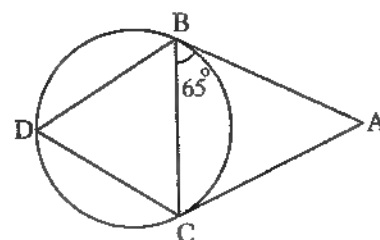


4 [a] In the opposite figure :

\overline{AB} and \overline{AC} are two tangent-segments to the circle at B and C

, $m(\angle ABC) = 65^\circ$

Find with proof : $m(\angle A)$ and $m(\angle D)$

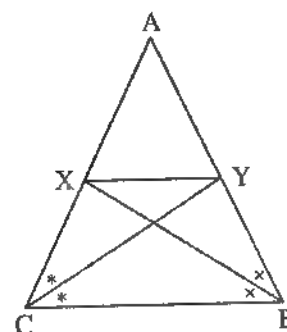


[b] In the opposite figure :

ABC is a triangle in which $AB = AC$, \overrightarrow{BX} bisects $\angle B$ and intersects \overline{AC} at X

, \overrightarrow{CY} bisects $\angle C$ and intersects \overline{AB} at Y

Prove that : The figure $BCXY$ is a cyclic quadrilateral.

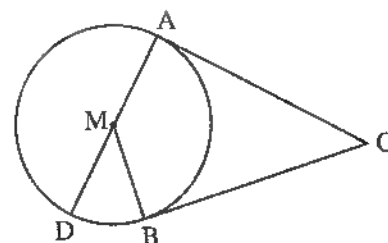


5 [a] In the opposite figure :

\overline{AD} is a diameter in a circle of centre M

, \overrightarrow{CA} and \overrightarrow{CB} are two tangents to the circle at A, B

Prove that : $m(\angle DMB) = m(\angle ACB)$



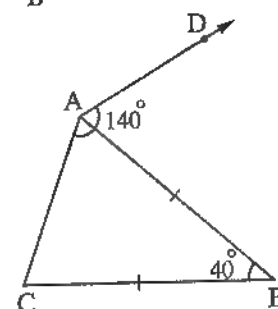
[b] In the opposite figure :

$BA = BC$, $m(\angle DAC) = 140^\circ$

and $m(\angle B) = 40^\circ$

Prove that :

\overrightarrow{AD} is a tangent to the circle passing through the vertices of $\triangle ABC$



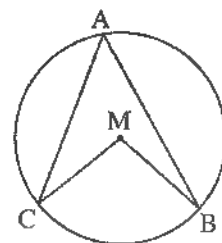
16 El-Menia Governorate



Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

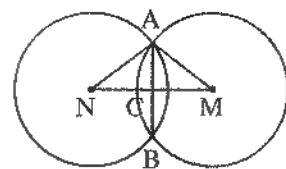
- (1) The two angles A and C in the right-angled triangle at B are
 (a) complementary. (b) supplementary.
 (c) adjacent. (d) vertically opposite angles.
- (2) The length of the opposite to the angle of measure 30° in the right-angled triangle is the length of the hypotenuse.
 (a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{2}$ (d) 2
- (3) The area of the rhombus whose diagonal lengths are 6 cm. , 8 cm. is cm^2
 (a) 2 (b) 14 (c) 24 (d) 48
- (4) The number of circles passing through three non-collinear points is
 (a) 1 (b) zero (c) 2 (d) 3
- (5) **In the opposite figure :**
 In the circle M ,
 if $m(\angle M) - m(\angle A) = 50^\circ$
 , then $m(\angle A) = \dots$
 (a) 40° (b) 50° (c) 100° (d) 130°
- (6) Which of the following shapes is a cyclic quadrilateral ?
 (a) rhombus (b) rectangle (c) parallelogram (d) trapezium



2 [a] In the opposite figure :

Two congruent circles M and N are intersecting at A and B
 If $MA = 10 \text{ cm.}$, $AB = 12 \text{ cm.}$

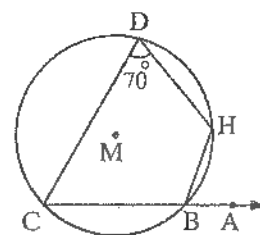
Find by proof : The length of \overline{MN}



[b] In the opposite figure :

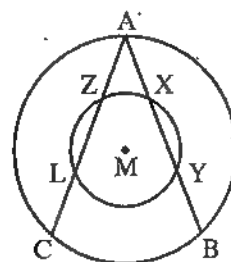
BCDH is a cyclic quadrilateral in the circle M
 , $m(\angle D) = 70^\circ$, $A \in \overrightarrow{CB}$, $m(\angle C) = \frac{1}{2} m(\angle H)$

Find by proof : $m(\angle ABH)$, $m(\angle H)$



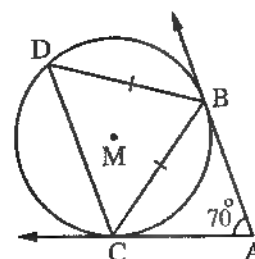
3 [a] In the opposite figure :

Two concentric circles at M
 $AB = AC$
Prove that : $XY = ZL$



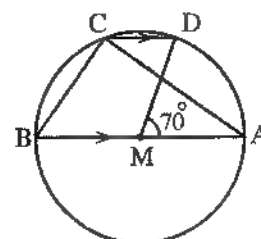
[b] In the opposite figure :

\overrightarrow{AB} , \overrightarrow{AC} are two tangents to the circle M
 $m(\angle BAC) = 70^\circ$, $BC = BD$
Find : $m(\angle ABD)$



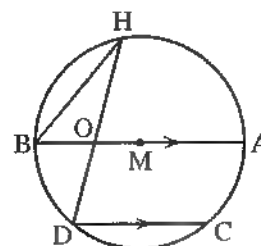
4 [a] In the opposite figure :

\overline{AB} is a diameter in the circle M
 $\overline{DC} \parallel \overline{AB}$, $m(\angle AMD) = 70^\circ$
Find by proof : $m(\angle ACD)$, $m(\angle ABC)$



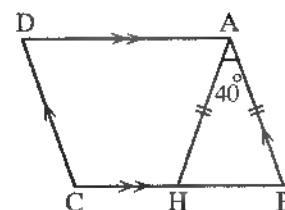
[b] In the opposite figure :

\overline{AB} is a diameter in the circle M
 $\overline{AB} \parallel \overline{DC}$, $m(\widehat{DC}) = 80^\circ$
 $m(\widehat{AH}) = 100^\circ$
Find by proof : $m(\angle DHB)$, $m(\angle AOH)$



5 In the opposite figure :

ABCD is a parallelogram
 $H \in \overline{BC}$ such that $AB = AH$, $m(\angle BAH) = 40^\circ$
(1) Find : $m(\angle AHB)$, $m(\angle D)$
(2) Prove that : AHCD is a cyclic quadrilateral.
(3) Prove that : \overrightarrow{AD} is a tangent to the circle passing through the vertices of $\triangle ABH$



Answer the following questions : (Calculator is allowed)

1 Choose the correct answer :

- (1) The chord which passes through the centre of the circle is called
 (a) tangent. (b) diameter. (c) radius. (d) side.

(2) The number of symmetry axes of a square

- (a) 2 (b) 3 (c) 4 (d) 5

(3) The inscribed angle which is opposite to the minor arc in a circle is

- (a) reflex. (b) right. (c) obtuse. (d) acute.

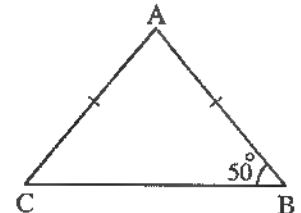
(4) In the opposite figure :

ABC is a triangle , $AB = AC$

, $m(\angle B) = 50^\circ$

, then $m(\angle A) = \dots\dots\dots$

- (a) 100° (b) 90° (c) 80° (d) 70°



(5) A tangent to a circle of diameter length 8 cm, is at a distance of cm. from its centre.

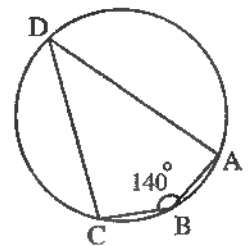
- (a) 4 (b) 3 (c) 8 (d) 6

(6) In the opposite figure :

$m(\angle B) = 140^\circ$

, then $m(\angle D) = \dots\dots\dots$

- (a) 40° (b) 60° (c) 30° (d) 50°



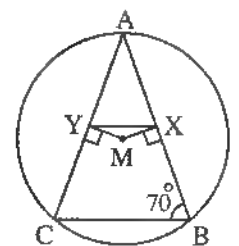
2 [a] In the opposite figure :

A circle M , $\overline{MX} \perp \overline{AB}$

, $\overline{MY} \perp \overline{AC}$, $m(\angle B) = 70^\circ$

(1) Prove that : $\overline{XY} \parallel \overline{BC}$

(2) Find with proof : $m(\angle YXM)$



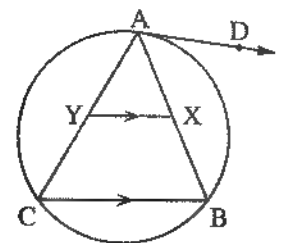
[b] In the opposite figure :

$\overline{XY} \parallel \overline{CB}$,

\overline{AD} is a tangent to the circle at A

Prove that :

\overline{AD} is a tangent to the circle passing through the points A , X and Y

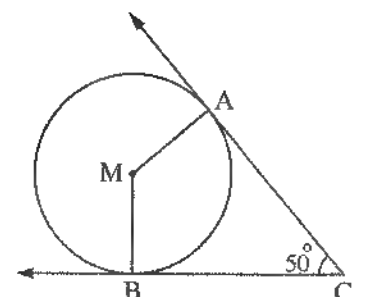


3 [a] In the opposite figure :

\overline{CA} , \overline{CB} are two tangents to the circle M

, $m(\angle C) = 50^\circ$

Find with proof : $m(\angle AMB)$



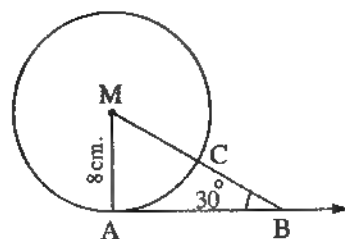
[b] In the opposite figure :

\overrightarrow{AB} is a tangent to the circle M at A and $MA = 8$ cm.

, $m(\angle ABM) = 30^\circ$

Find : (1) The length of \overline{MB}

(2) $m(\widehat{CA})$

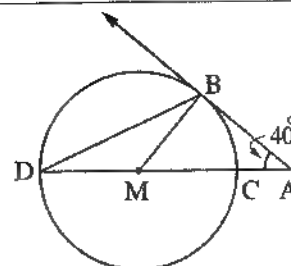


4 [a] In the opposite figure :

\overrightarrow{AB} is a tangent to the circle at B , $m(\angle A) = 40^\circ$

, \overrightarrow{AM} intersects the circle M at C and D

Find with proof : $m(\angle BDC)$



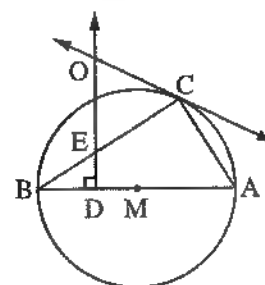
[b] In the opposite figure :

\overline{AB} is a diameter in the circle M

, \overrightarrow{CO} is a tangent to the circle at C and $\overrightarrow{DO} \perp \overline{AB}$

Prove that : (1) ADEC is a cyclic quadrilateral.

(2) $OE = OC$

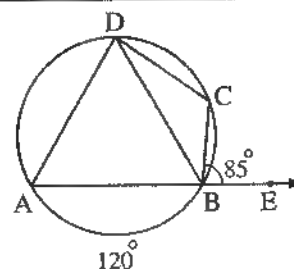


5 [a] In the opposite figure :

$E \in \overline{AB}$, $E \notin \overline{AB}$

, $m(\widehat{AB}) = 120^\circ$, $m(\angle CBE) = 85^\circ$

Find : $m(\angle BDC)$



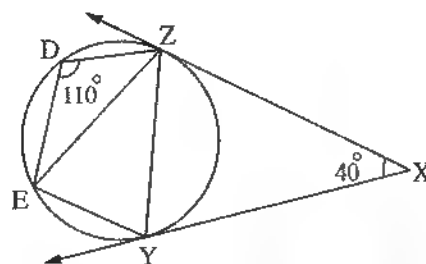
[b] In the opposite figure :

\overrightarrow{XY} , \overrightarrow{XZ} are two tangents to the circle

from the point X , $m(\angle X) = 40^\circ$

, $m(\angle D) = 110^\circ$

Prove that : $m(\widehat{ZE}) = m(\widehat{ZY})$



Answer the following questions : (Calculator is allowed)

1 Choose the correct answer :

(1) The two tangents which are drawn from the two endpoints of a diameter of a circle are

(a) parallel. (b) equal in length. (c) congruent. (d) intersecting.

(2) The number of the axes of symmetry in the equilateral triangle =

(a) 1 (b) 2 (c) 3 (d) an infinite number.

(3) M and N are two intersecting circles, their radii lengths are 5 cm. , 2 cm. , then $MN \in \dots\dots\dots$

- (a) $[3, 7]$ (b) $[3, 7[$ (c) $]3, 7]$ (d) $]3, 7[$

(4) The number of common tangents of two distant circles is

- (a) 1 (b) 2 (c) 3 (d) 4

(5) The length of side opposite to the angle of measure 30° in the right-angled triangle equals the length of the hypotenuse.

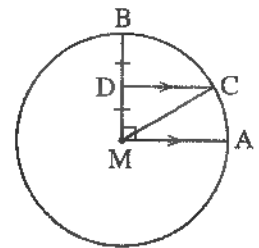
- (a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

(6) In the opposite figure :

$\overline{AM} \parallel \overline{CD}$, $MD = DB$

, $m(\angle AMB) = 90^\circ$, then $m(\widehat{AC}) = \dots\dots\dots$

- (a) 45° (b) 60°
(c) 30° (d) 90°



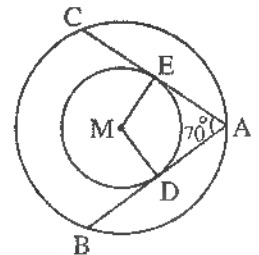
2 [a] Find the measure of the arc which represents $\frac{1}{2}$ its circle , then calculate the length of this arc if the length of the radius is 7 cm. ($\pi = \frac{22}{7}$)

[b] In the opposite figure :

Two concentric circle at M , \overline{AB} and \overline{AC} are two tangents to the smaller circle at D and E , $m(\angle A) = 70^\circ$

(1) Find : $m(\angle DME)$

(2) Prove that : $AB = AC$

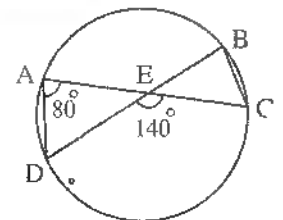


3 [a] In the opposite figure :

$m(\angle CED) = 140^\circ$

, $m(\angle A) = 80^\circ$

Find : $m(\angle C)$

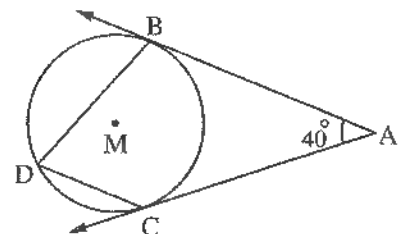


[b] In the opposite figure :

\overline{AB} and \overline{AC} are two tangents to the circle at B and C

, $m(\angle A) = 40^\circ$

Find with proof : $m(\angle D)$

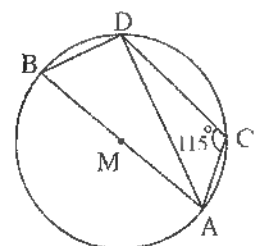


4 [a] In the opposite figure :

\overline{AB} is a diameter of the circle M ,

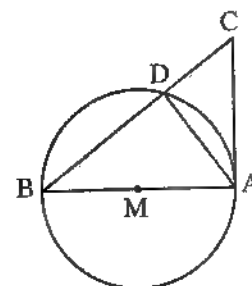
$m(\angle ACD) = 115^\circ$

Find with proof : $m(\angle DAB)$



[b] In the opposite figure :

\overline{AB} is a diameter of the circle M
 \overline{AC} is a tangent touches it at A
 if $AC = 9$ cm. and $BM = 6$ cm.
Find : The lengths of \overline{BC} and \overline{AD}



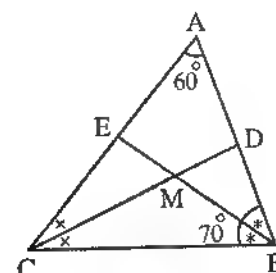
5 [a] State three cases of cyclic quadrilateral.

[b] In the opposite figure :

$m(\angle A) = 60^\circ$, \overline{BE} bisects $\angle ABC$
 $m(\angle B) = 70^\circ$, \overline{CD} bisects $\angle ACB$

(1) Find : $m(\angle BMC)$

(2) Prove that : $ADME$ is a cyclic quadrilateral.



19

Qena Governorate



Answer the following questions : (Calculators are Permitted)

1 Choose the correct answer :

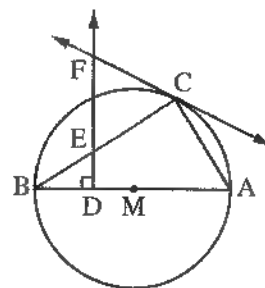
- (1) If the area of the circle $M = 16\pi \text{ cm}^2$, A is a point on its plane where $MA = 8$ cm.
 , then A is
 (a) outside the circle. (b) inside the circle.
 (c) on the circle. (d) on the centre of the circle.
- (2) A tangent to a circle of diameter length 6 cm. is at distance of cm. from its centre.
 (a) 6 (b) 12 (c) 3 (d) 2
- (3) The centre of the circumcircle of the triangle is the intersection point of its
 (a) altitudes of triangle. (b) medians of a triangle.
 (c) perpendicular bisectors of the sides of a triangle. (d) bisectors of its angles.
- (4) The inscribed angle drawn in a semicircle is angle.
 (a) acute. (b) obtuse. (c) right. (d) straight.
- (5) The two tangent-segments drawn from a point outside a circle are
 (a) equal in length. (b) not equal in length.
 (c) perpendicular. (d) parallel.
- (6) The figure is said to be cyclic quadrilateral if the measure of any exterior angle at any vertex equal to of the interior angle at the opposite vertex.
 (a) the measure. (b) half the measure.
 (c) twice the measure. (d) third the measure.

2 [a] In the opposite figure :

\overline{AB} is a diameter in the circle M
 \overrightarrow{CF} is a tangent to the circle at C , $\overrightarrow{DE} \perp \overline{AB}$

Prove that :

- (1) ADEC is a cyclic quadrilateral.
- (2) $FE = FC$

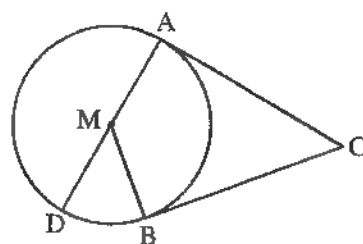


[b] The length of \overline{AB} is 4 cm. , draw a circle of radius length 3 cm. and passes through the two points A , B how many circles can be drawn ? Find the radius length of the smallest circle that can be drawn to pass through the two points A , B

3 [a] In the opposite figure :

\overline{AD} is a diameter in the circle M
 \overrightarrow{CA} and \overrightarrow{CB} are two tangents to the circle M
 at A and B respectively

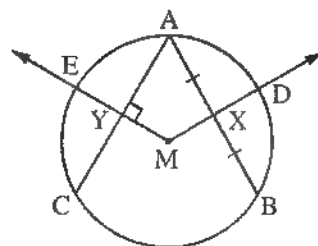
Prove that : $m(\angle DMB) = m(\angle ACB)$



[b] In the opposite figure :

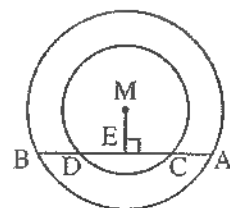
\overline{AB} and \overline{AC} are two equal chords in length in circle M
 and X is the midpoint of \overline{AB} , \overrightarrow{MX} intersects the circle at D
 $\overrightarrow{MY} \perp \overline{AC}$ intersects it at Y and intersects the circle at E

Prove that : $XD = YE$



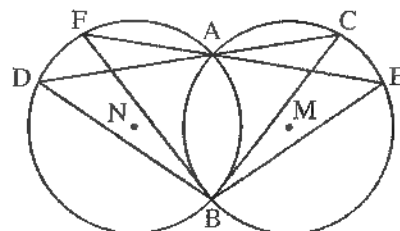
4 [a] In the opposite figure :

Two concentric circles M
 \overline{AB} is a chord in the larger circle intersecting the smaller circle at C and D , $\overrightarrow{ME} \perp \overline{AB}$ **Prove that :** $AC = BD$



[b] In the opposite figure :

M and N are two intersecting circles at A and B
 \overrightarrow{AC} intersects the circle M at C
 and intersects the circle N at D ,
 \overrightarrow{AE} intersects the circle M at E
 and intersects the circle N to F
Prove that : $m(\angle EBC) = m(\angle FBD)$



5 ABC is an acute-angled triangle drawn inside a circle , draw $\overrightarrow{AD} \perp \overline{BC}$

to cut \overline{BC} at D and cuts the circle at E , then draw $\overrightarrow{CN} \perp \overline{AB}$ to cut \overline{AB} at N

Porve that : (1) ANDC is a cyclic quadrilateral. (2) $m(\angle BND) = m(\angle BED)$

20 Luxor Governorate



Answer the following questions :

1 Choose the correct answer :

(1) The sum of measures of the accumulative angles at a point =°

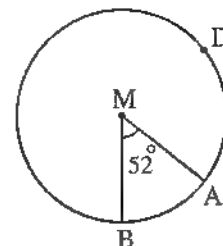
- (a) 80 (b) 120 (c) 360 (d) 630

(2) In the opposite figure :

If $m(\angle AMB) = 52^\circ$

, then $m(\widehat{ADB}) = \dots\dots\dots^\circ$

- (a) 52 (b) 104 (c) 128 (d) 308



(3) The length of side opposite to the angle of measure 30° in the right-angled triangle equals the hypotenuse length.

- (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{\sqrt{3}}{2}$ (d) 2

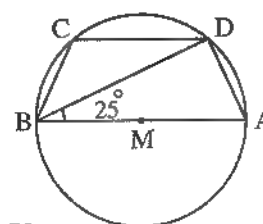
(4) In the opposite figure :

\overline{AB} is a diameter in the circle M

, $m(\angle ABD) = 25^\circ$

, then $m(\angle C) = \dots\dots\dots$

- (a) 50° (b) 100° (c) 115° (d) 125°



(5) The sum of lengths of any two sides of a triangle the length of the third side.

- (a) < (b) > (c) = (d) \leq

(6) The number of circles pass by three non-collinear points =

- (a) infinite number. (b) 3 (c) 1 (d) 0

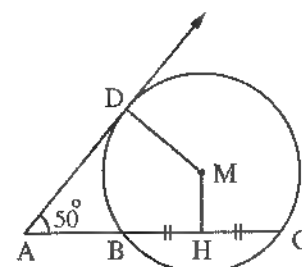
2 [a] In the opposite figure :

\overline{AD} is a tangent to the circle at D ,

H is the midpoint of \overline{BC}

, $m(\angle A) = 50^\circ$

Find with proof : $m(\angle DMH)$

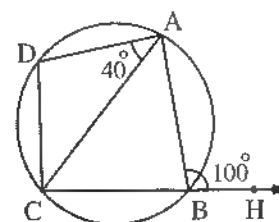


[b] In the opposite figure :

$m(\angle ABH) = 100^\circ$

, $m(\angle DAC) = 40^\circ$

Prove that : $m(\widehat{CD}) = m(\widehat{AD})$

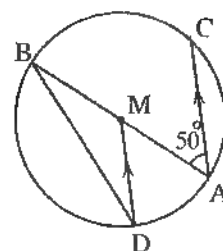


3 [a] In the opposite figure :

\overline{AB} is a diameter in the circle M

, $\overline{AC} \parallel \overline{MD}$, $m(\angle CAB) = 50^\circ$

Find : $m(\angle MDB)$



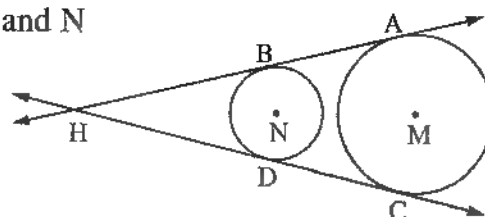
[b] In the opposite figure :

\overleftrightarrow{AH} and \overleftrightarrow{CH} are two tangents to the two circles M and N

touch the circle M at A and C

touch the circle N at B and D

Prove that : $AB = CD$

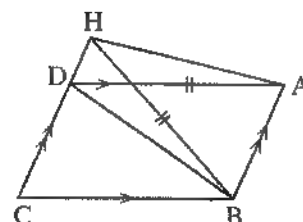


4 [a] In the opposite figure :

ABCD is a parallelogram $H \in \overline{CD}$

where $BH = AD$

prove that : ABDH is a cyclic quadrilateral.



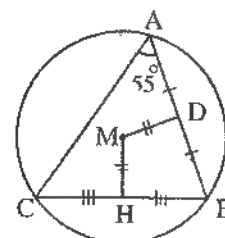
[b] In the opposite figure :

D is the midpoint of \overline{AB}

, H is the midpoint of \overline{BC} ,

$m(\angle A) = 55^\circ$, $MD = MH$

Find : $m(\angle B)$



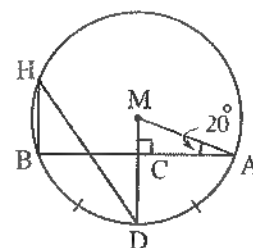
5 [a] In the opposite figure :

$\overleftrightarrow{MC} \perp \overline{AB}$ and intersects the circle M at D

which is the midpoint of \widehat{AB}

, $m(\angle MAB) = 20^\circ$

Find : (1) $m(\widehat{AD})$ (2) $m(\angle DHB)$

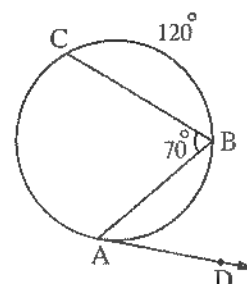


[b] In the opposite figure :

\overleftrightarrow{AD} is a tangent to the circle at A

, $m(\angle B) = 70^\circ$, $m(\widehat{BC}) = 120^\circ$

Find : $m(\angle BAD)$





Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from the given ones :

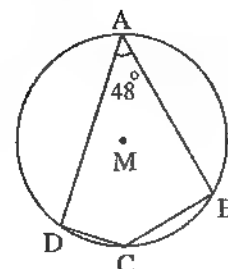
(1) In the opposite figure :

$m(\angle A) = 48^\circ$, then

the measure of major arc $\widehat{BD} = \dots\dots\dots$

- (a) 260° (b) 265° (c) 264°

(d) 262°



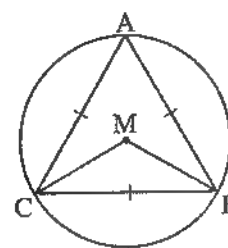
(2) In the opposite figure :

ABC is an equilateral triangle inscribed in circle M

, then $m(\angle BMC) = \dots\dots\dots$

- (a) 50° (b) 120° (c) 60°

(d) 100°



(3) In the opposite figure :

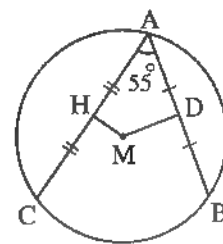
D is the midpoint of \overline{AB} , H is the midpoint of \overline{AC}

, $m(\angle A) = 55^\circ$

, then $m(\angle DMH) = \dots\dots\dots$

- (a) 120° (b) 130° (c) 135°

(d) 125°



(4) Number of axes of symmetry of the circle = $\dots\dots\dots$

- (a) zero (b) one (c) infinite number. (d) 4

(5) The length of side opposite to the angle of measure 30° in the right-angled triangle equals $\dots\dots\dots$ the length of the hypotenuse.

- (a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$ (c) $\sqrt{2}$ (d) 2

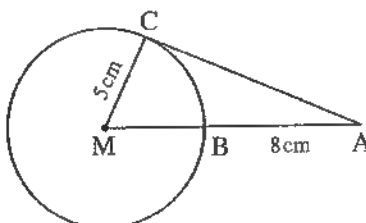
(6) In the opposite figure :

\overline{AC} is a tangent to circle M at C

if $MC = 5$ cm. , $AB = 8$ cm.

, then $AC = \dots\dots\dots$ cm.

- (a) 5 (b) 10 (c) 13



(d) 12

2 [a] M and N are two circles of radii length 9 cm. and 4 cm. respectively.

Show the position of each of them with respect to the other if :

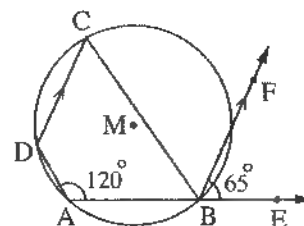
- (1) $MN = 5$ cm. (2) $MN = 10$ cm.

[b] In the opposite figure :

ABCD is a quadrilateral inscribed in circle M

, $\overrightarrow{BF} \parallel \overrightarrow{DC}$, $m(\angle EBF) = 65^\circ$, $m(\angle BAD) = 120^\circ$

Find : $m(\angle ADC)$

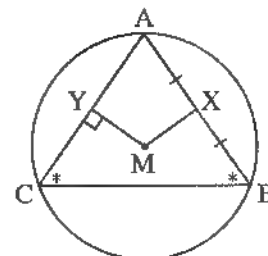


3 [a] In the opposite figure :

ABC is a triangle inscribed in circle M,

$m(\angle B) = m(\angle C)$, X is the midpoint of \overline{AB} , $\overline{MY} \perp \overline{AC}$

Prove that : $MX = MY$

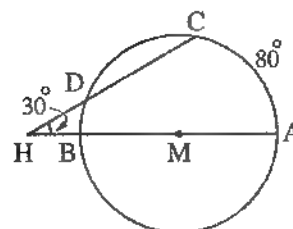


[b] In the opposite figure :

\overline{AB} is a diameter in circle M, $\overrightarrow{AB} \cap \overrightarrow{CD} = \{H\}$,

$m(\angle AHC) = 30^\circ$, $m(\widehat{AC}) = 80^\circ$

Find : $m(\widehat{CD})$



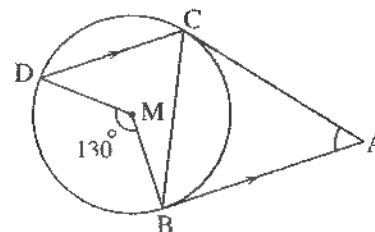
4 [a] In the opposite figure :

\overline{AB} and \overline{AC} are two tangent-segments to the circle M

at B and C, $\overline{AB} \parallel \overline{CD}$, $m(\angle BMD) = 130^\circ$

(1) Find : $m(\angle ABC)$

(2) Prove that : \overline{CB} bisects $\angle ACD$

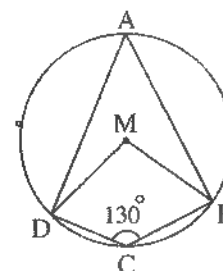


[b] In the opposite figure :

In the circle M,

if $m(\angle BCD) = 130^\circ$

Find : $m(\angle BMD)$



5 [a] In the opposite figure :

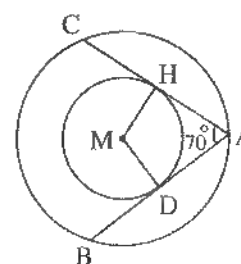
Two concentric circles at M

\overline{AB} and \overline{AC} are two tangent-segments to smaller circle at D and H

, $m(\angle BAC) = 70^\circ$

Prove that : (1) $AB = AC$

(2) Find : $m(\angle DMH)$



[b] In the opposite figure :

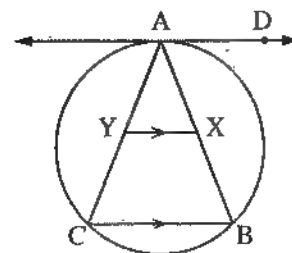
ABC is a triangle inscribed in a circle ,

\overleftrightarrow{AD} is a tangent to a circle at A

, $X \in \overline{AB}$, $Y \in \overline{AC}$, $\overline{XY} \parallel \overline{BC}$

Prove that :

\overleftrightarrow{AD} is a tangent to the circle which passes through the points A , X , Y



22 South Sinai Governorate



Answer the following questions :

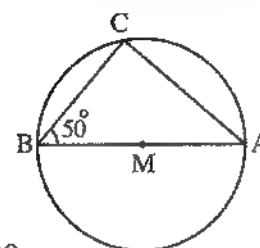
1 Choose the correct answer from the given ones :

(1) In the opposite figure :

\overline{AB} is a diameter in the circle M

$m(\angle ABC) = 50^\circ$, then $m(\widehat{BC}) = \dots\dots\dots^\circ$

- (a) 40 (b) 50 (c) 80 (d) 100



(2) The rhombus in which the lengths of diagonals are 6 cm. and 8 cm. its area = $\dots\dots\dots \text{cm}^2$

- (a) 12 (b) 14 (c) 24 (d) 48

(3) If M is a circle of radius length r cm. , then the length of the semicircle = $\dots\dots\dots \text{cm}$.

- (a) $2\pi r$ (b) $\frac{1}{4}\pi r$ (c) $\frac{1}{2}\pi r$ (d) πr

(4) The longest chord in the circle is called $\dots\dots\dots$

- (a) diameter. (b) tangent. (c) secant. (d) radius.

(5) The image of the point (2 , 3) by rotation R (O , 180°) is the point $\dots\dots\dots$

- (a) (2 , 3) (b) (-2 , 3) (c) (2 , -3) (d) (-2 , -3)

(6) The sum of measures of the two opposite angles in the cyclic quadrilateral equal $\dots\dots\dots^\circ$

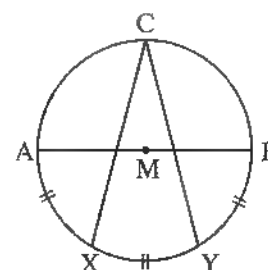
- (a) 180 (b) 120 (c) 100 (d) 30

2 [a] In the opposite figure :

\overline{AB} is a diameter in the circle M

, the length of $(\widehat{AX}) =$ the length of $(\widehat{XY}) =$ the length of (\widehat{BY})

find with proof : $m(\angle C)$



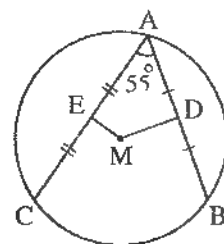
[b] In the opposite figure :

\overline{AB} and \overline{AC} are two chords in the circle M

, D is the midpoint of \overline{AB} and E is the midpoint of \overline{AC} ,

$m(\angle BAC) = 55^\circ$

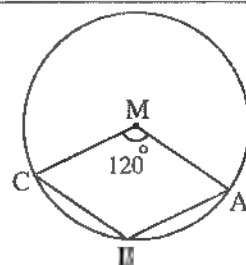
Find with proof : $m(\angle DME)$



3 [a] In the opposite figure :

M is a circle and $m(\angle AMC) = 120^\circ$

Find with proof : $m(\angle ABC)$



[b] Two circles M and N with radii lengths of 7 cm. and 4 cm. respectively

Show the position of each of them respect to the other in the following cases :

(1) $MN = 8$ cm.

(2) $MN = 3$ cm.

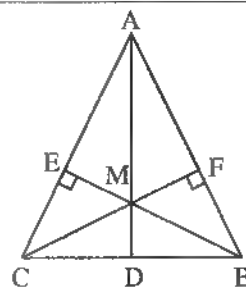
(3) $MN = 12$ cm.

4 [a] In the opposite figure :

$\triangle ABC$, $\overline{BE} \perp \overline{AC}$, $\overline{CF} \perp \overline{AB}$

$\overrightarrow{AM} \cap \overrightarrow{BC} = \{D\}$

Prove that : MDCE is a cyclic quadrilateral.

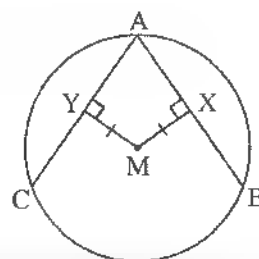


[b] In the opposite figure :

M is a circle, \overline{AB} and \overline{AC} are two chords,

$\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$, $AB = 6$ cm., $MX = MY$

Find with proof : The length of \overline{AY}



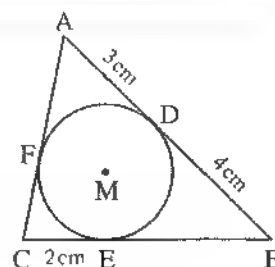
5 [a] In the opposite figure :

M is an inscribed circle in the triangle ABC

and touches its sides at D, E and F

, $AD = 3$ cm., $CE = 2$ cm., $BD = 4$ cm.

Find with proof : The perimeter of $\triangle ABC$

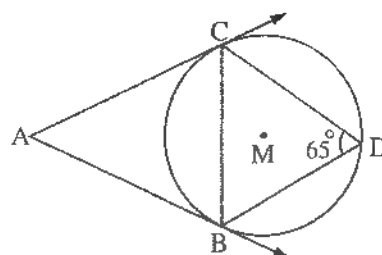


[b] In the opposite figure :

\overrightarrow{AB} and \overrightarrow{AC} are two tangents of the circle M

, $m(\angle D) = 65^\circ$

Find with proof : $m(\angle A)$



23 Red Sea Governorate



Answer the following questions :

1 Choose the correct answer from the given ones :

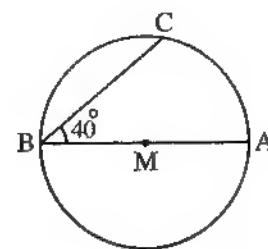
- (1) Number of the circles that pass through three non-collinear points equals
 (a) zero (b) one (c) three (d) an infinite number

(2) In the opposite figure :

\overline{AB} is a diameter in the circle M

, $m(\angle ABC) = 40^\circ$, then $m(\widehat{BC}) = \dots\dots\dots$

- (a) 40° (b) 50°
 (c) 90° (d) 100°



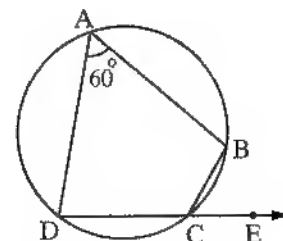
- (3) If the two circles M and N are touching externally, their radii lengths are 9 cm. , r cm. , and $MN = 14$ cm. , then $r = \dots\dots\dots$ cm.

- (a) 5 (b) 7 (c) 10 (d) 23

(4) In the opposite figure :

If $m(\angle BAD) = 60^\circ$, then $m(\angle BCE) = \dots\dots\dots$

- (a) 30° (b) 60°
 (c) 80° (d) 120°



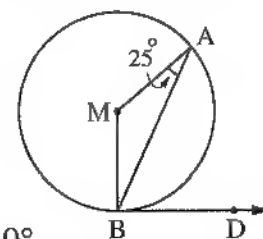
(5) In the opposite figure :

If \overline{BD} is a tangent to the circle M

, $m(\angle BAM) = 25^\circ$

, then $m(\angle ABD) = \dots\dots\dots$

- (a) 25° (b) 50° (c) 65° (d) 120°



- (6) Circumference of a circle is 6π cm. , L is a straight line at a distance of 3 cm. from its centre , then L is

- (a) a tangent to the circle. (b) a secant to the circle.
 (c) outside the circle. (d) the diameter to the circle.

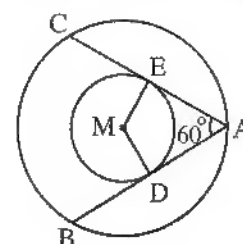
2 [a] In the opposite figure :

Two concentric circles M ,

\overline{AB} , \overline{AC} are two tangents to the smaller circle , $m(\angle A) = 60^\circ$

(1) Find : $m(\angle DME)$

(2) Prove that : $AB = AC$

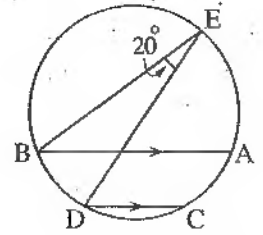


[b] In the opposite figure :

\overline{AB} , \overline{CD} are two parallel chords

, $m(\angle BED) = 20^\circ$

Find : $m(\widehat{AC})$



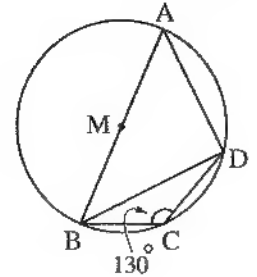
3 [a] In the opposite figure :

ABCD is a quadrilateral inscribed in a circle M

where $M \in \overline{AB}$

, $m(\angle BCD) = 130^\circ$

Find : $m(\angle A)$, $m(\angle ABD)$



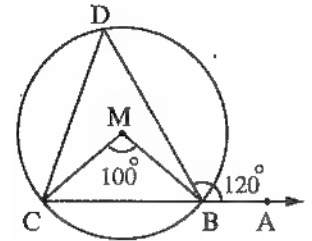
[b] In the opposite figure :

In the circle M :

$m(\angle BMC) = 100^\circ$

, $m(\angle ABD) = 120^\circ$

Find with proof : $m(\angle DCB)$

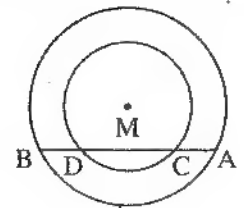


4 [a] In the opposite figure :

Two concentric circle M

, \overline{AB} is a chord in the large circle intersecting the small circle at C and D

Prove that : $AC = BD$

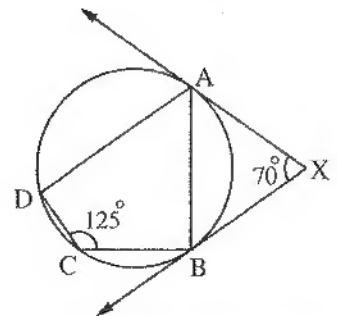


[b] In the opposite figure :

\overrightarrow{XA} and \overrightarrow{XB} are two tangents to a circle at A and B

, $m(\angle AXB) = 70^\circ$, $m(\angle DCB) = 125^\circ$

Prove that : \overline{AB} bisects $\angle DAX$

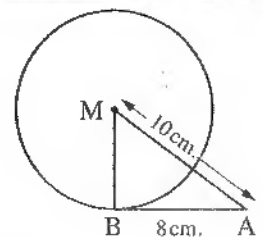


5 [a] In the opposite figure :

\overline{AB} is a tangent to a circle M at B

, $AB = 8$ cm. , $AM = 10$ cm.

Find : The area of $\triangle ABM$



[b] ABC is a triangle inscribed in a circle , \overleftrightarrow{BD} is a tangent to the circle at B

, $X \in \overline{AB}$, $Y \in \overline{BC}$ where $\overline{XY} \parallel \overline{BD}$

Prove that : AXYC is a cyclic quadrilateral.

24 Matrouh Governorate



Answer the following questions : (Calculator is allowed)

1 Choose the correct answer :

- (1) The perimeter of the square whose area is 81 cm^2 is
 (a) 24 cm. (b) 8 cm. (c) 9 cm. (d) 36 cm.
- (2) The two opposite angles in the cyclic quadrilateral are
 (a) equal. (b) complementary. (c) supplementary. (d) alternate.
- (3) ABC is a triangle where $(AB)^2 = (AC)^2 + (BC)^2$, $m(\angle B) = 40^\circ$, then $m(\angle A) = \dots\dots\dots$
 (a) 40° (b) 50° (c) 90° (d) 130°
- (4) The measure of the arc which represents $\frac{1}{3}$ the measure of the circle equals
 (a) 60° (b) 90° (c) 120° (d) 240°
- (5) The area of the triangle whose base length is 10 cm. and its height is 6 cm.
 equals cm^2 .
 (a) 6 (b) 10 (c) 30 (d) 60
- (6) If the two circles M, N are touching internally, the radius length of one of them is 3 cm.
 , and $MN = 8 \text{ cm}$, then the radius length of the other circle equals
 (a) 5 cm. (b) 6 cm. (c) 11 cm. (d) 12 cm.

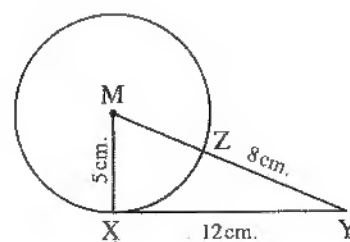
2 [a] In the opposite figure :

M is a circle whose radius length is 5 cm.

, $XY = 12 \text{ cm}$, $\overline{MY} \cap \text{the circle M} = \{Z\}$

and $ZY = 8 \text{ cm}$.

Prove that : \overleftrightarrow{XY} is a tangent to the circle M at X



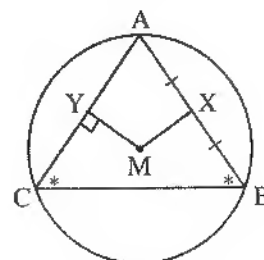
[b] In the opposite figure :

$\triangle ABC$ is inscribed in the circle M

, in which $m(\angle B) = m(\angle C)$

, X is the midpoint of \overline{AB} , $\overline{MY} \perp \overline{AC}$

Prove that : $MX = MY$



- 3 [a] Prove that :** The measure of the angle of tangency is equal to the measure of the inscribed angle subtended by the same arc.

[b] ABCD is a quadrilateral drawn in a circle, $F \in \overline{AB}$

, draw $\overrightarrow{FE} \parallel \overline{CB}$ to cut \overline{CD} at E, $\overrightarrow{DF} \cap \overline{CB} = \{X\}$

Prove that : (1) AFED is a cyclic quadrilateral.

(2) $m(\angle BXF) = m(\angle EAD)$

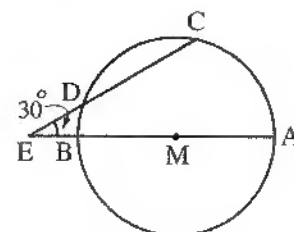
4 [a] In the opposite figure :

\overline{AB} is a diameter in the circle M

, $\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$

, $m(\angle AEC) = 30^\circ$, $m(\widehat{AC}) = 80^\circ$

Find : $m(\widehat{CD})$

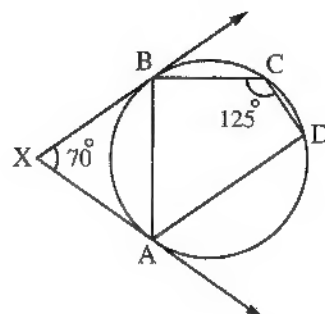


[b] In the opposite figure :

\overrightarrow{XA} and \overrightarrow{XB} are two tangents to the circle at A and B

, $m(\angle AXB) = 70^\circ$, $m(\angle DCB) = 125^\circ$

Prove that : \overline{AB} bisects $\angle DAX$



5 [a] Mention three cases of the cyclic quadrilateral.

[b] In the opposite figure :

ABCD is a quadrilateral inscribed in the circle M

where $M \in \overline{AB}$, $CB = CD$

, $m(\angle BCD) = 140^\circ$

Find : (1) $m(\angle A)$

(2) $m(\angle D)$

